1. Find the coefficient of x^2 in the expansion of $(2x-1)^7$.

Let $a = 2x$; $b = -1$ $\quad \binom{n}{r} a^{n-r} b^r$

$$\binom{7}{5}(2x)^2(-1)^5$$

$$= 21 \cdot 4x^2 \cdot -1$$

$$= -84x^2$$

2.~~3~~. Find the coefficient of x^6 in the expansion of $\left(2x - \frac{1}{2}\right)^{10}$.

Let $a = 2x$; $b = -\frac{1}{2}$ $\quad \binom{n}{r} a^{n-r} b^r$

$$\binom{10}{4}(2x)^6\left(-\frac{1}{2}\right)^4$$

$$= 210 \cdot 64x^6 \cdot \frac{1}{16}$$

$$= 840x^6$$

~~$~~3. Find the value of c for which the coefficient of x^4 in the expansion of $(2x+c)^7$ is 70.

$$\binom{7}{3}(2x)^4(c)^3$$

$$35 \cdot 16\cancel{x^4} \cdot c^3 = 70$$

$$560c^3 = 70$$

$$\therefore c^3 = \frac{1}{8}$$

$$\therefore c = \frac{1}{2}$$

4 ~~3~~. (a) Find k such that the equation $2x^2 + kx + 2k = 0$ has exactly one solution.

For one solution $b^2 - 4ac = 0$

$$k^2 - 4(2)(2k) = 0$$

$$k^2 - 16k = 0$$

$$k(k - 16) = 0$$

$$\therefore k = 0 \quad \text{or} \quad k = 16$$

5 ~~4~~. Find the value of "a" if the equations $2x + 3y = 6$ and $6x + ay = 9$ are

(i) parallel (ii) perpendicular

$$2x + 3y = 6 \iff y = -\frac{2}{3}x + 2$$

$$6x + ay = 9 \iff y = -\frac{6}{a}x + \frac{9}{a}$$

(i) parallel $\Rightarrow -\frac{2}{3} = \frac{-6}{a}$

$$\iff a = 6 \cdot \frac{3}{2} = 9$$

(ii) perpendicular $\Rightarrow -\frac{2}{3} \times \frac{-6}{a} = -1$

$$\iff \frac{4}{a} = -1$$

$$\therefore a = -4$$

6 ~~8~~. The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the value(s) of "a".

$$\binom{7}{2}(x)^5\left(\frac{1}{ax^2}\right)^2$$

$$= 21 \cdot x^5 \cdot \frac{1}{a^2x^4}$$

$$= \frac{21}{a^2}$$

$$\therefore \frac{21}{a^2} = \frac{7}{3} \Leftrightarrow a^2 = 21 \times \frac{3}{7} = 9$$

$$\Leftrightarrow a = \pm 3$$

7 ~~8~~. (a) Find the equation of the line which passes through both the intersection of $x + y = 2$ and $2x + 3y = 8$ and the point $(0, 0)$

$x + y = 2$ — eq. 1

$2x + 3y = 8$ - eq. 2

eq. 2 - 2 × eq. 1 : $y = 4$

sub into eq. 1 : $x = -2$

$\therefore$ intersect at $(-2, 4)$

$\therefore$ equation of line is $y - 0 = \frac{4}{-2}(x - 0) \Rightarrow y = -2x$

7. (b) Simplify $\dfrac{x^3-y^3}{x^3+y^3} \times \dfrac{(x-y)^2+xy}{x^2+xy+y^2}$

$$= \frac{(x-y)(x^2+xy+y^2)}{(x+y)(x^2-xy+y^2)} \times \frac{x^2-xy+y^2}{x^2+xy+y^2}$$

$$= \frac{x-y}{x+y}$$

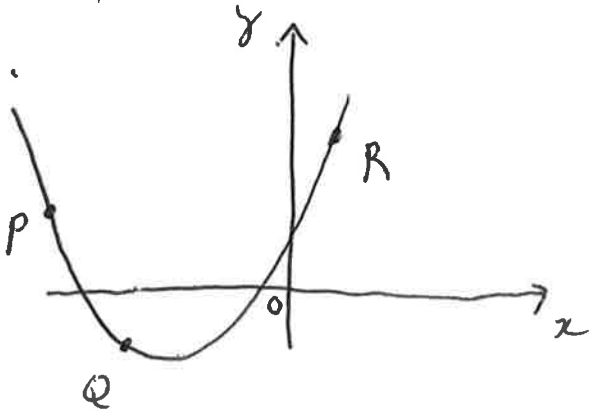
8. Find the term of x^5 in the expansion $(1+2x)^8$.

$$\binom{8}{5}(1)^3(2x)^5$$

$$= 56 \cdot 1 \cdot 32x^5$$

$$= 1792x^5$$

9. The function f is given by $f(x) = ax^2 + bx + c$. Part of the graph of f is shown.



The graph of f passes through the points $P(-10, 12)$, $Q(-5, -3)$ and $R(5, 27)$.

Find the values of a, b and c.

$Q(-5,-3) \Rightarrow (-3) = a(-5)^2 + b(-5) + c$

$\Leftrightarrow -3 = 25a - 5b + c$ eq. 1

$R(5,27) \Rightarrow (27) = a(5)^2 + b(5) + c$

$\Leftrightarrow 27 = 25a + 5b + c$ eq. 2

eq. 1 − eq 2

$\Rightarrow -10b = -30$

$\therefore b = 3$

$P(-10,12) \Rightarrow (12) = a(-10)^2 + b(-10) + c$

$\Leftrightarrow 12 = 100a - 10b + c$ eq. 3

eq. 3 − eq. 1

$\Rightarrow 75a - 15 = 15$

$\therefore a = \frac{2}{5}$

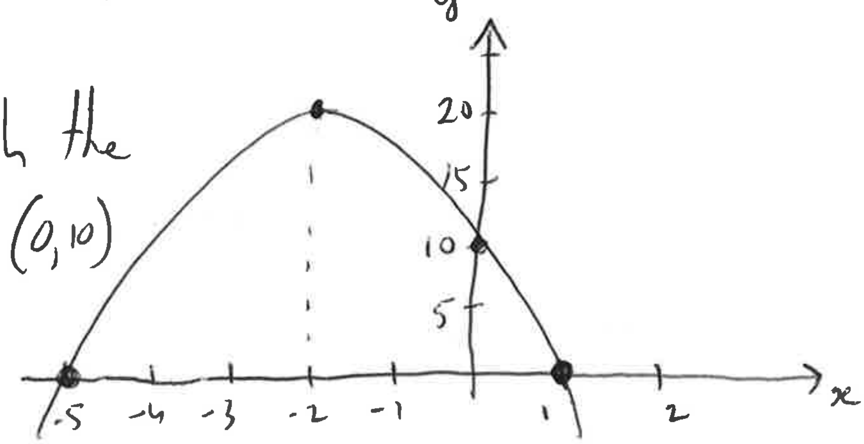
sub $a = \frac{2}{5}$ and $b = 3$ into eq. 1

$\Rightarrow c = 2$

$\therefore y = \frac{2}{5}x^2 + 3x + c$

10 ~~8~~. Let $f(x) = a(x-p)(x-q)$. Part of the graph of f is shown.

The graph passes through the points $(-5,0)$, $(1,0)$ and $(0,10)$



(a) Write down the value of 'p' and 'q'.

(b) Find the value of 'a'.

(a) $p = -5$

$q = 1$

(b) $y = a(x+5)(x-1)$

sub in $(0, 10)$

$10 = a((0)+5)((0)-1)$

$\therefore -5a = 10$

$\therefore a = -2$

11 ~~4~~. Let $f(x) = a(x+3)^2 - 6$

(a) Write down the coordinates of the vertex of the graph of f.

(b) Given that $f(1) = 2$, find the value of 'a'.

(c) Hence find the value of $f(3)$

(a) $(-3,-6)$

(b) $2 = a((1)+3)^2 - 6$

$2 = 16a - 6$

$\therefore a = \frac{1}{2}$

(c) $f(3) = \frac{1}{2}(3+3)^2 - 6$

$= 12$

12 ~~8~~. The equation $x^2 + 2kx + 3 = 0$ has two equal real roots. Find the possible values of 'k'.

$a = 1$; $b = 2k$; $c = 3$

$(2k)^2 - 4(1)(3) = 0$

$4k^2 - 12 = 0$

$4(k^2 - 3) = 0$

$k^2 = 3$

$\therefore k = \pm\sqrt{3}$

13 ~~8~~. Let $f(x) = 2x^2 + 12x + 5$.

(a) Write the function f, giving your answer in the form $f(x) = a(x-h)^2 + k$.

(b) The graph of g is formed by translating the graph of f by 4 units in the positive x-direction and 8 units in the positive y-direction. Find the coordinates of the vertex of the graph of g.

(a) $f(x) = 2\left[x^2 + 6x + 9 - 9\right] + 5$

$= 2\left[(x+3)^2 - 9\right] + 5$

$= 2(x+3)^2 - 18 + 5$

$= 2(x+3)^2 - 13$

(b) $g(x) = 2(x-1)^2 - 5$

14~~8~~. The height, h metres above the water, of a stone thrown off a bridge is modeled by the function $h(t) = 15t + 20 - 4.9t^2$, where t is the time in seconds after the stone is thrown.

(a) What is the initial height from which the stone is thrown?

(b) What is the maximum height reached by the stone?

(c) For what length of time is the height of the stone greater than 20m?

(d) How long does it take for the stone to hit the water below the bridge?

(a) initial height is when $t = 0$, $\therefore h = 20$ m

(b) maximum hieght is 31.5 m

(c) 3.06 s

(d) 4.07 s

15. (a) Find the common ratio for the geometric series

$$\frac{1}{12} + \frac{1}{8} + \frac{3}{16} + \dots$$

(b) Hence, find the least value of n such that

$S_n > 800$

(a) $r = \frac{3}{2}$ or 1.5

(b) $800 < \dfrac{\frac{1}{12}\left(1 - \frac{3}{2}^n\right)}{1 - \frac{3}{2}}$

$-4800 < 1 - \frac{3}{2}^n$

$\frac{3}{2}^n > 4801$

$\therefore n \approx 20.9$

$\therefore n = 21$

16. For a geometric progression with $u_3 = 24$ and $u_6 = 3$, find S_∞

$$S_\infty = \frac{u_1}{1-r}$$

$$3 = u_1 r^5$$
$$24 = u_1 r^2$$

$$\frac{1}{8} = r^3$$

$$\therefore r = \frac{1}{2}$$

$$24 = u_1\left(\frac{1}{2}\right)^2$$

$$\therefore u_1 = 96$$

$$\therefore S_\infty = \frac{96}{1-\frac{1}{2}} = 192$$

17. In a geometric sequence, the first term is 3 and the sixth term is 96.

(a) Find the common ratio

(b) Find the least values of n such that $u_n > 3000$

(a) $96 = 3(r)^5$

$32 = r^5$

$\therefore r = 2$

(b) $3000 < 3(2)^{n-1}$

$1000 < 2^{n-1}$

$\therefore n - 1 = 10$

$\therefore n = 11$

18. In an arithmetic sequence, the first term is 28 and the common difference is 50. In a geometric sequence, the first term is 1 and the common ratio is 1.5. Find the least value of n such that the nth term of the geometric sequence is greater than the nth term of the arithmetic sequence.

$A.P = 28 + (n-1)50$ $\qquad$ $G.P. = 1(1.5)^{n-1}$

$= 50n - 22$

$\Rightarrow 50n - 22 < 1(1.5)^{n-1}$

$50n - 1.5^{n-1} < 22$ $\qquad$ Using GDC, $n = 18$

19. Find the term in x^4 in the expansion of $\left(\frac{x}{2}-3\right)^7$

$\binom{7}{3}\left(\frac{x}{2}\right)^4(-3)^3$

$= 35 \cdot \frac{1}{16}x^4 \cdot -27$

$= \frac{-945}{16}x^4$

20. Consider the arithmetic sequence 3, 7, 11, 15, ...

(a) Write down the common difference

(b) Find u_{71}

(c) Find the value of n such that $u_n = 99$

(a) $d = 4$

(b) $u_{71} = 3 + (71-1)4$

$= 283$

(c) $99 = 3 + (n-1)4$

$99 = 4n - 1$

$\therefore 4n = 100$

$\therefore n = 25$

21. The first three terms of an infinite geometric sequence are 64, 16, and 4.

(a) Write down the value of r.

(b) Find u_4

(c) Find the sum to infinity of this sequence.

(a) $r = \frac{1}{4}$ or 0.25

(b) $u_4 = 64\left(\frac{1}{4}\right)^3$

$= 1$

(c) $S_\infty = \frac{u_1}{1-r} \Rightarrow \frac{64}{1-\frac{1}{4}} \Rightarrow \frac{256}{3}$

22. In an arithmetic sequence, $u_6 = 25$ and $u_{12} = 49$

(a) Find the common difference

(b) Find the first term of the sequence

(a) $49 = u_1 + (12-1)d$

$49 = u_1 + 11d$ – eq. 1

$25 = u_1 + (6-1)d$

$25 = u_1 + 5d$ – eq. 2

$\therefore$ $24 = 6d$

$\therefore d = 4$

(b) $49 = u_1 + 11(4)$

$49 = u_1 + 44$

$\therefore u_1 = 5$

23. Consider the arithmetic sequence $22, x, 38, \ldots$

(a) Find the value of x

(b) Find u_{31}

(a) $38 - x = x - 22$

$\therefore 2x = 60$

$\therefore x = 30$

(b) $u_{31} = 22 + (31-1)8$

$= 262$

24. Find the x^3 term in the expansion of $(2x+3)^5$

$\binom{5}{2}(2x)^3(3)^2$

$= 10 \cdot 8x^3 \cdot 9$

$= 720x^3$

25. Consider the arithmetic sequence $3, 4.5, 6, 7.5, \ldots$

(a) Find u_{63}

(b) Find the value of n such that $S_n = 840$

(a) $u_{63} = 3 + (63-1)1.5$

$= 96$

(b) $S_n = 840$

$\Rightarrow 840 = \frac{n}{2}\left(2(3) + (n-1)(1.5)\right)$

$\Rightarrow 1680 = n(1.5n + 4.5) \Rightarrow 1.5n^2 + 4.5n - 1680 = 0$

$\therefore n = 35$