Trigonometry Review (Topic 3)

Triangles

You need to know SOH CAH TOA (for right angle trig only)

You need to know the values of sine, cosine, and tangent for the following angles:

Angle ^{o}	0	30	45	60	90
Rad					
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					

Oblique (non-right) triangles

Sine law - use when you know "a pair" (a side and its opposite angle)

ideal for finding sides:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

ideal for finding angles

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Remember that when you use sine law to find an angle, you must consider both an acute and an obtuse solution (your GDC only gives the acute solution).

Cosine law - use to find a third side when you know two sides and the included angle

-use to find an angle when you know all three sides

ideal for finding sides:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

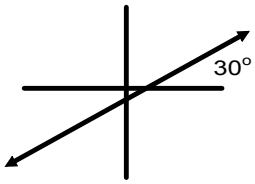
ideal for finding angles

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Lines in 2-D

A line that passes through the x-axis and forms some angle θ with the positive x-axis has a slope given by:

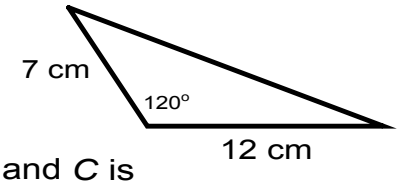
$m = \tan\theta$



Triangle Area

$A = \frac{1}{2}bh$

$A = \frac{1}{2}ab\sin C$

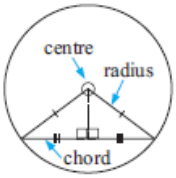


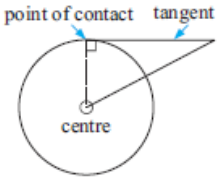
where *a* and *b* are side lengths and *C* is the included angle

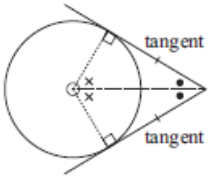
Circles, chords, tangents, sector areas, and segments

CHORDS AND TANGENTS

Right angled triangles occur in chord and tangent problems.

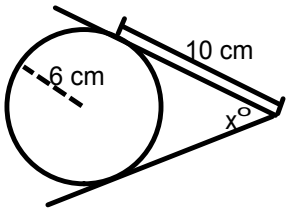




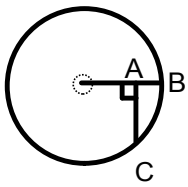


Find the unknowns for:

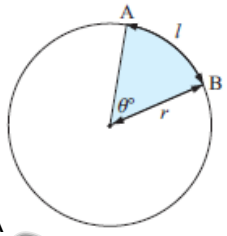
1.



2. In the given figure AB = 1cm and AC = 3cm. Find:
   a) the radius of the circle
   b) the angle subtended by chord BC at the centre of the circle.



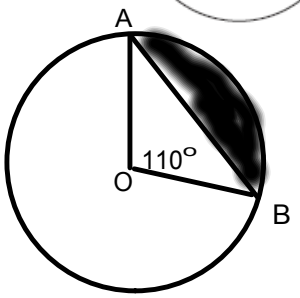
SECTORS



- $l = \left(\frac{\theta}{360}\right) \times 2\pi r$
- $\text{area} = \left(\frac{\theta}{360}\right) \times \pi r^2$

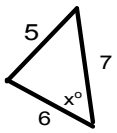
These formulae are given in terms of radians in your booklet

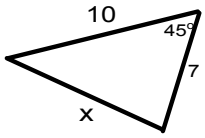
Find

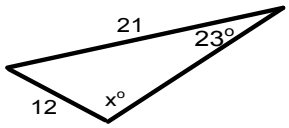


a) the area of ΔOAB
b) the area of sector OAB
c) the area of the shaded segment

Find the indicated side or angle: (diagrams not drawn to scale)





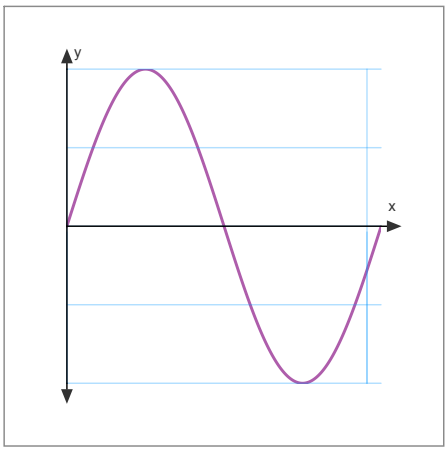


Radian measure: (YOUR CALCULATOR DEFAULTS TO RADIANS)

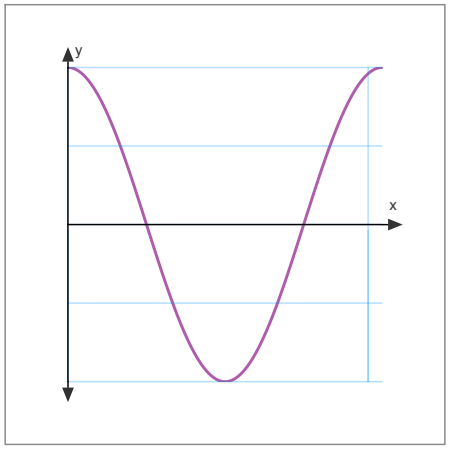
π radians = 180°

Basic graphs:

$y = \sin x$

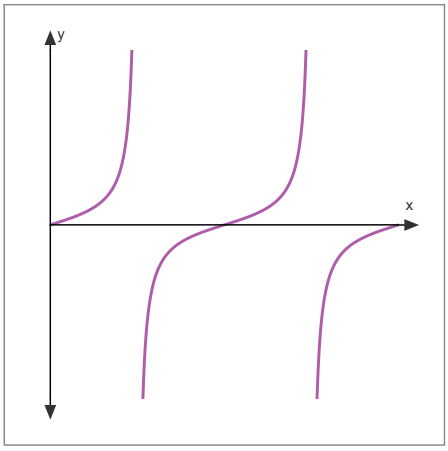


$y = \cos x$



Properties:
-Amplitude is 1 (half the vertical distance travelled by the graph)
-Principal axis is on the x-axis (line y=0)
-Period is 2π
-Basic sine graph starts in the middle and moves up
-Basic cosine graph starts "high" and moves down

$y = \tan x$



Properties:
-vertical asymptotes at $\frac{\pi}{2}$, $\frac{3\pi}{2}$, ...
(this is because tangent is undefined at these values)
-periodic with a period of π

Modelling with sine and cosine graphs

$y = A\sin(B(x-C)) + D$ $\qquad$ $y = A\cos(B(x-C)) + D$

**please note that the letter choices are arbitrary.

The "*A*" value controls the amplitude. It may be found by taking the difference between the max and min, then dividing by 2. It also controls reflections through the x-aixs. A negative "*A*" value will refelct the basic graph in the x-axis.

The "*B*" value controls the period. If *B* = 1, then the period is 2π.

Generally the period of a graph is:

$$Period = \frac{2\pi}{B}$$

Also the "*B*" value can be found by:

$$B = \frac{2\pi}{period}$$

The "*C*" value controls the horizontal shift of the graph.

The graph of $y = \cos(x - \frac{\pi}{4})$ would be a cosine graph shifted

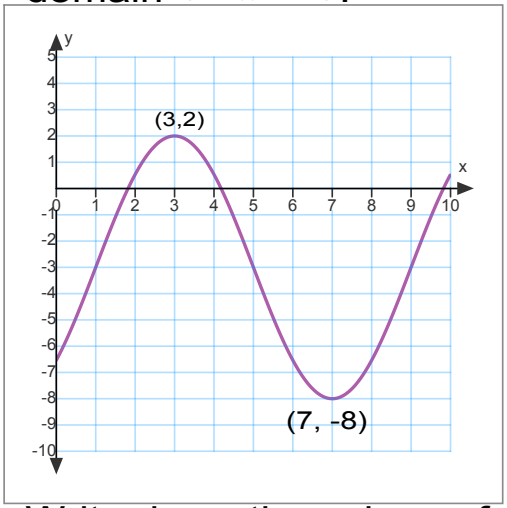
_____ units _____.

The graph of $y = \sin(x + \pi)$ would be a sine graph shifted

_____ units _____.

The "*D*" value controls the vertical shift and the location of the principal axis. It is halfway between the max and min values of the graph.

The graph of $f(x) = p\cos(q(x-r)+s$ is shown below for the domain $0 \le x \le 10$.



Write down the values of:

p

q

r

s

Hence, find all solutions to the equation $f(x) = -6$

The equation f(x) = *k* has exactly one solution. Write down the two possible values of *k*.

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Trigonometric Equations

A working knowledge of the trig ratios from 0° to 360° is required. If you know the values on the table on page 1 of this handout, then the unit circle should be easy.

Solve $2\sin\left(\frac{x}{4}\right)=\sqrt{3}$ for $0\le x\le 10\pi$

Sometimes factoring is required.

Solve $2\sin^2 x-\sin x=1$ for $0\le x\le 2\pi$

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Trig Identities

These appear in your formula booklet and may be re-arranged.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$

Double angle formulae:

$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Tangent equations (recall that the default tangent function has a period of π).

Solve $\tan\left(x - \frac{\pi}{4}\right) = \sqrt{3}$ for all x.

Solve $3\sin x + \sqrt{3}\cos x = 0$ for $0 \le x \le 2\pi$

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$ for $0 \le x \le 2\pi$

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