Topic 3 - Circular Trigonometry Workbook

<u>Angles between 0 and 360 degrees</u>

1. Set your GDC to degree mode.

   In the graph menu set the *x*-window from 0 to 90, and the *y* from -3 to 3.

   Draw the graph of y=sin$x$. Note if the y values are positive or negative.

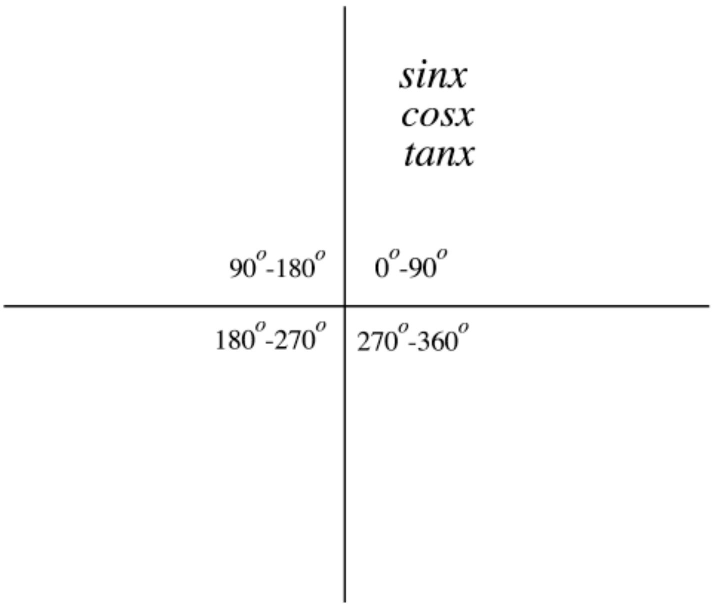
   Do the same for graphs of y=cos$x$ and y=tan$x$.

   Now repeat your 3 graphs using an *x*-window from 90 to 180; repeat again for an *x*-window from 180 to 270; and finally for an *x*-window from 270 to 360.

   Use your answers to complete the table below.

	0 to 90	**90 to 180**	**180 to 270**	**270 to 360**
y=sin$x$				
y=cos$x$				
y=tan$x$				

2. On the diagram below write which give positive values. From 0 to 90 degrees has been completed for you.



3. Re-set your GDC window to 0 to 360 for x, and -1.1 to 1.1 for y.

   Draw a graph of y=sin$x$ and y=0.8.
   Find the points of intersection using your GDC to solve sin$x$=0.8.
   Add the two angles together.
   Repeat using y=sin$x$ *and* y=a, where a is a value between 0 and 1.

4. Draw a graph of y=sin$x$ and y=(-0.5).
   Find the points of intersection using your GDC to solve sin$x$=(-0.5).
   Is there any relationship between your two answers?
   Repeat using y=sin$x$ *and* y=a, where a is a value between 0 and -1.

5. Repeat 3 and 4, but use y=cos$x$.

6. Repeat 3 and 4, but use y=tan$x$, and adjust your y-window to -5 to 5.

Use you answers and finding from above to find **two** values for x for each of the following:

i) sin x = 0.5

ii) cos x = 0.5

iii) tan x = 0.5

iv) sin x = (-0.866)

v) cos x = (-0.65)

vi) tan x = 0.3

vii) tan x =(-1.73)

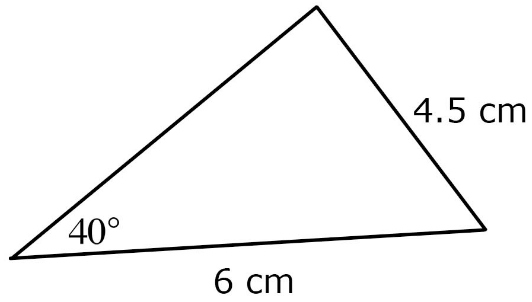
viii) cos x = (-0.32)

Sine and Cosine Laws

1 In triangle *ABC*, *AB*=6 cm, *BC*=8 cm, and *AC*=5 cm.
Find the angle *ABC*.

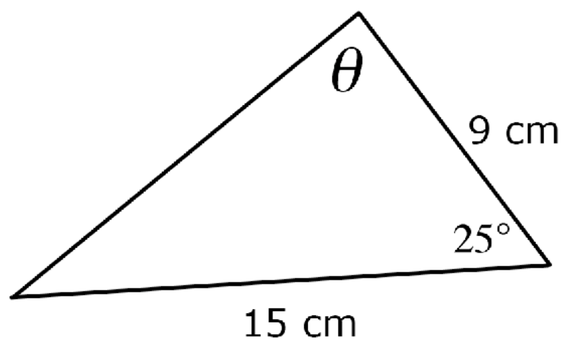
2 Find the two possible areas of the triangle ABC shown in the diagram below.

Not to scale



3 In the triangle ABC, $ABC = 42°$, $AB = 5$ cm and $BC = 7.8$ cm.
Find the length of AC.

4



a) Find the value of $\sin\theta$ in the triangle above.

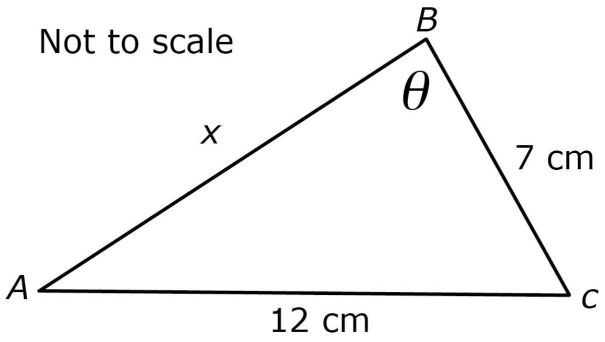
b) Find the value(s) of θ in the triangle above.

5 In triangle PQR, $PQ = 32$ cm, $QR = 50$ cm and $QPR = 80°$.

a) Find the angle QRP.

b) Find PR.

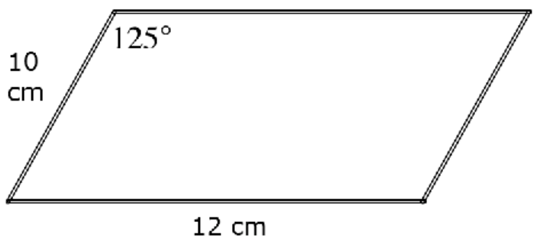
6



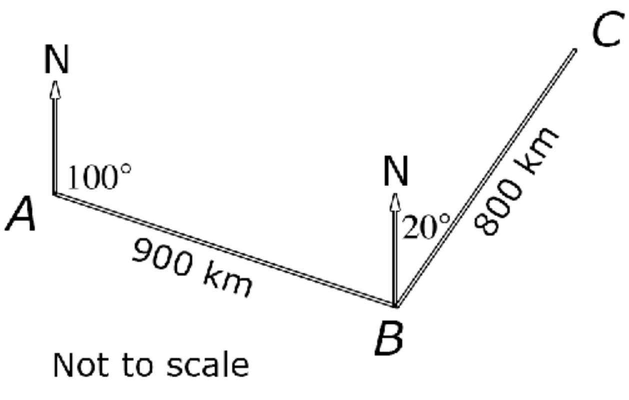
$\sin\theta = 0.98481$

Find two values of x.

7 Find the area of the parallelogram shown in the diagram below.



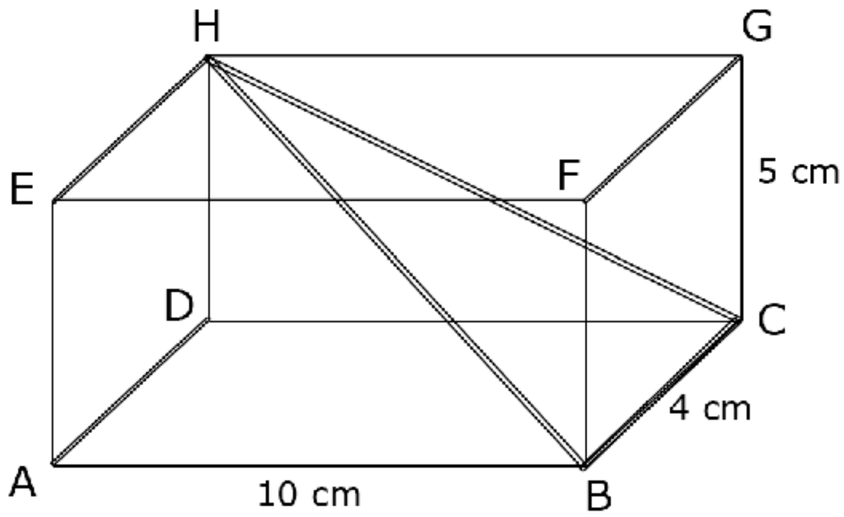
8 A plane leaves an airfield, *A*, on a bearing of 100°, flying to point *B* a direct distance of 900 km. At *B* the plane takes a new bearing of 20°, and flies direct to point *C*, a distance of 800 km. At *C* the plane turns and flies directly back to the airfield.
These details are shown in the diagram below.



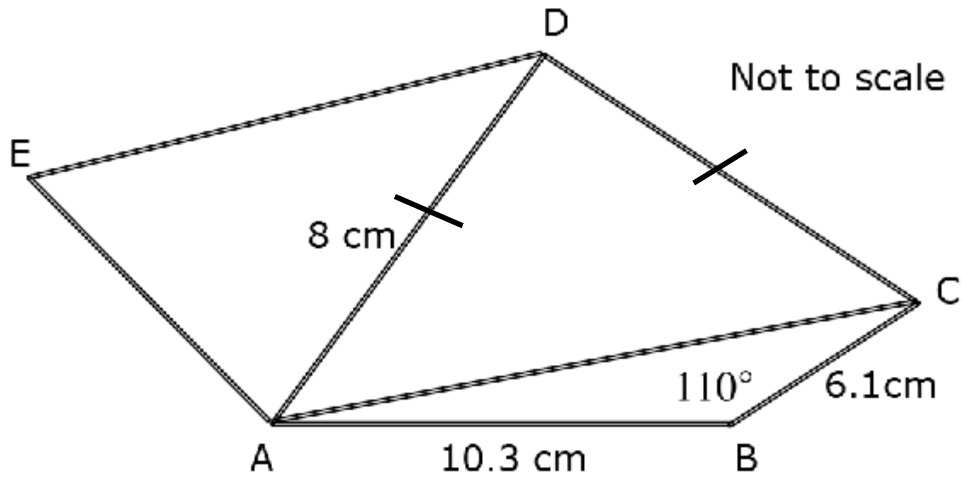
a) Find the direct distance from *C* to *A*.

b) Find the bearing the plane flies on from *C* to *A*.

9 A cuboid *ABCDEFGH* is shown below. Find the angle *BHC*.



10 The diagram below shows a pentagon ABCDE, with *AB*=10.3 cm, *BC*=6.1 cm, *CD*=5.5 cm, *AD*=8 cm, and angle ABC=110^o.



a) Find *AC*.

b) Find *ACD*.

c) Given *BAE*=126^o, find *DAE*.

d) Given that the area of *AED*=18.7 cm^2, find *AE*.

e) Find the area of the pentagon.

<u>Radian Measure</u>

1. Convert each of the following into radians, giving your answer in terms of pi.

   a) 120^o c) 45^o

   b) 270^o d) 330^o

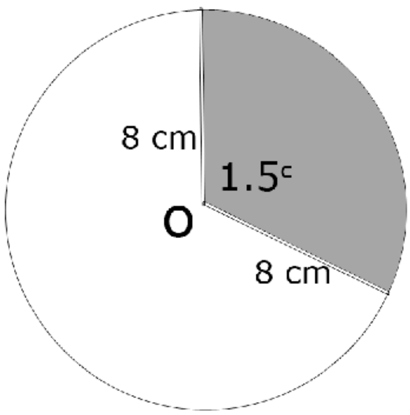
2. Convert each of the following radians into degrees.

   a) $\dfrac{\pi}{3}$ c) $\dfrac{5\pi}{3}$

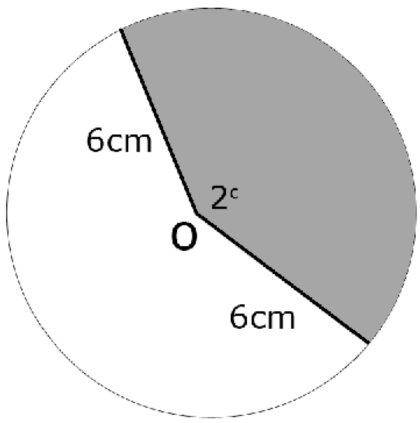
   b) $\dfrac{3\pi}{4}$ d) $\dfrac{7\pi}{6}$

3. Calculate the length of the minor arc in each of the shapes below. The angle is marked in radians.

   a)

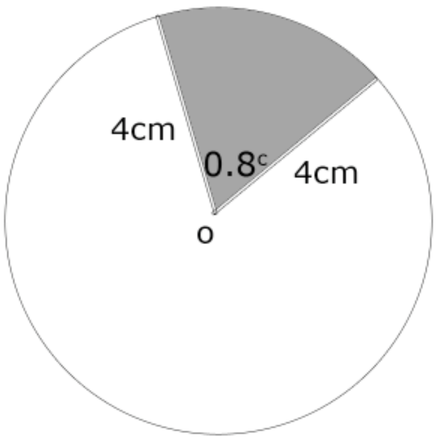


   b)

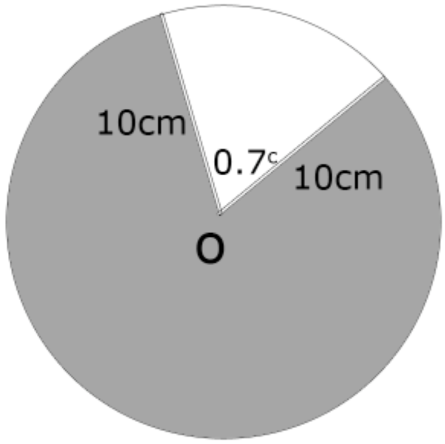


4. Calculate the area of the shaded part in each of the shapes below. The angle is marked is radians.

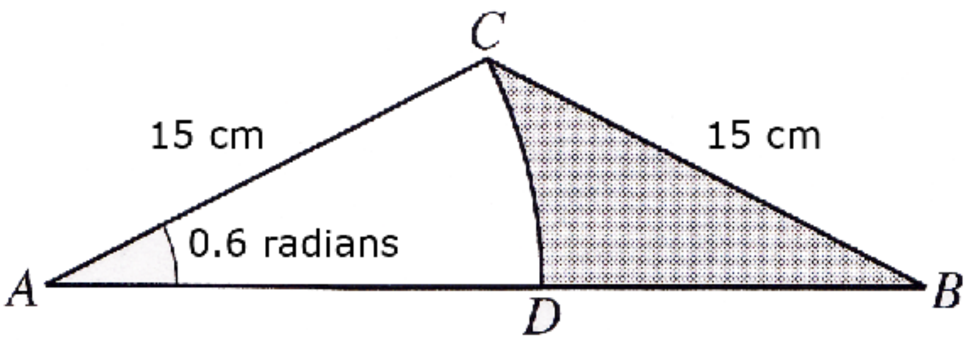
a)



b)



5.



The diagram shows an isosceles triangle ABC in which BC = AC = 15 cm, and angle BAC = 0.6 radians. Dc is an arc of a circle, centre A. Find, correct to 1 decimal place,

a) the area of the shaded region,

b) the perimeter of the shaded region.

Unit Circle

The following should be completed without the use of tables or a GDC.

1 Given that $\sin\theta = \frac{1}{2}$ and $0 \le \theta \le 2\pi$, find:

a) 2 values of θ,

b) the value(s) of $\cos\theta$.

2 In the following $0 \le \theta \le 2\pi$.

a) Given that $\sin\theta = \frac{\sqrt{3}}{2}$ and θ is obtuse find θ.

b) Given that $\tan\theta = \sqrt{3}$ and θ is reflex find θ.

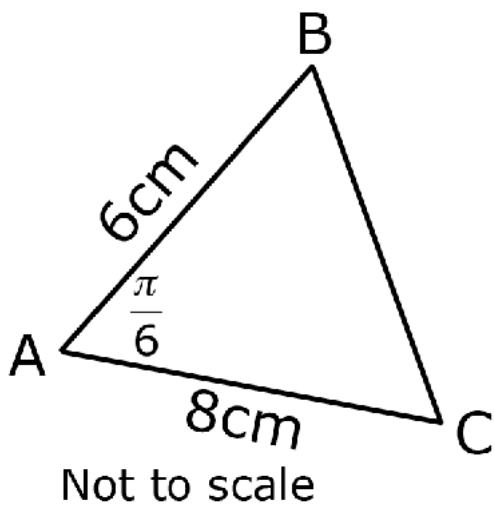
c) Given that $\cos\theta = -\frac{1}{2}$ and θ is obtuse find θ.

d) Given that $\tan\theta = \frac{1}{\sqrt{3}}$ and θ is reflex find θ.

e) Given that $\cos\theta = \frac{1}{2}$ and θ is reflex find θ.

f) Given that $\sin\theta = \frac{1}{2}$ and θ is acute find θ.

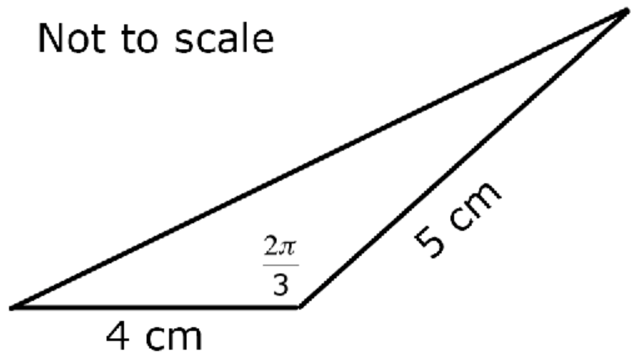
3 The triangle, ABC, drawn opposite shows AB = 6 cm, AC = 8 cm and angle BAC = $\frac{\pi}{6}$.



a) Calculate the area of triangle *ABC*.

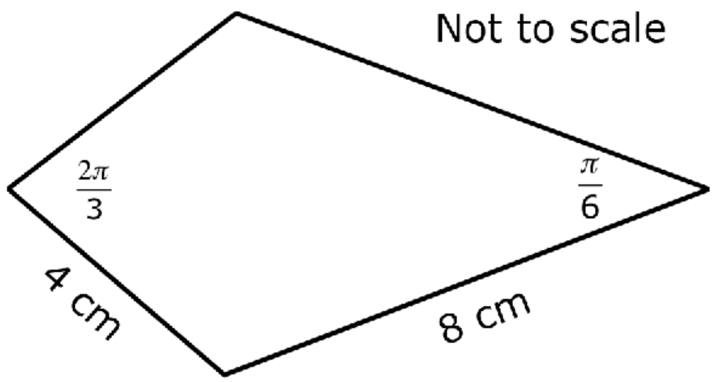
b) Find *BC*, giving your answer as an exact number.

4 Calculate the missing side in the triangle below, giving your answer as a surd.

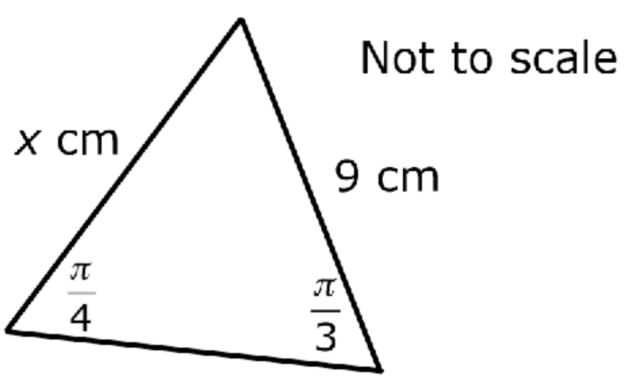


For question 5 assume that the kite is symmetrical

5 The diagram below shows a kite. Find the area of the kite.



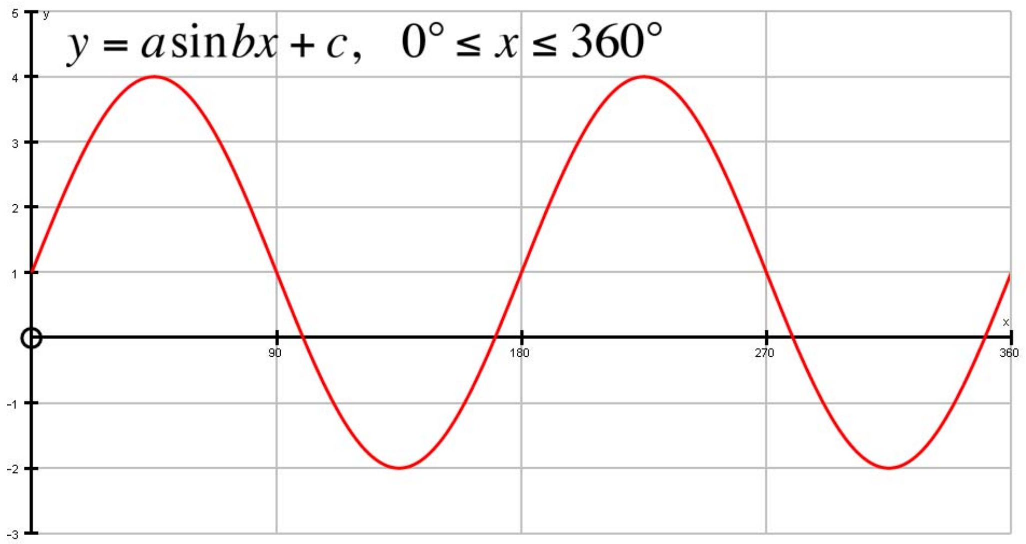
6 Find the exact value of x in the diagram below.



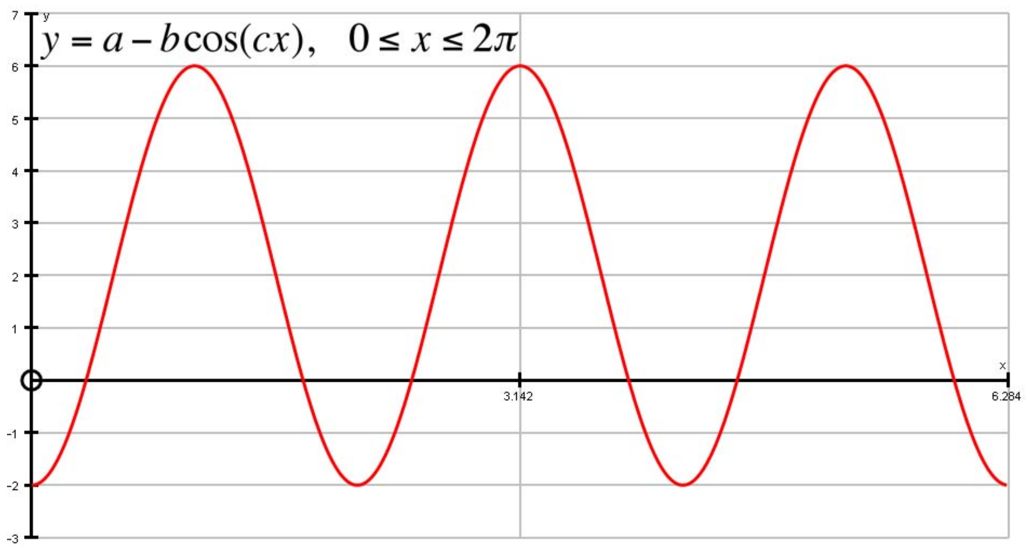
Trigonometric Models

1 Find the values of a, b, and c in each of these graphs below.

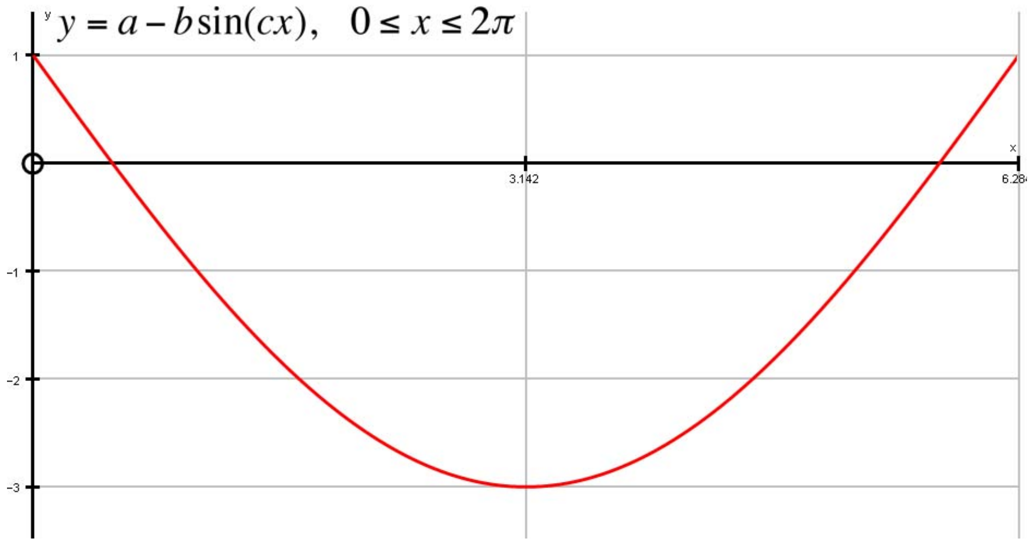
a)



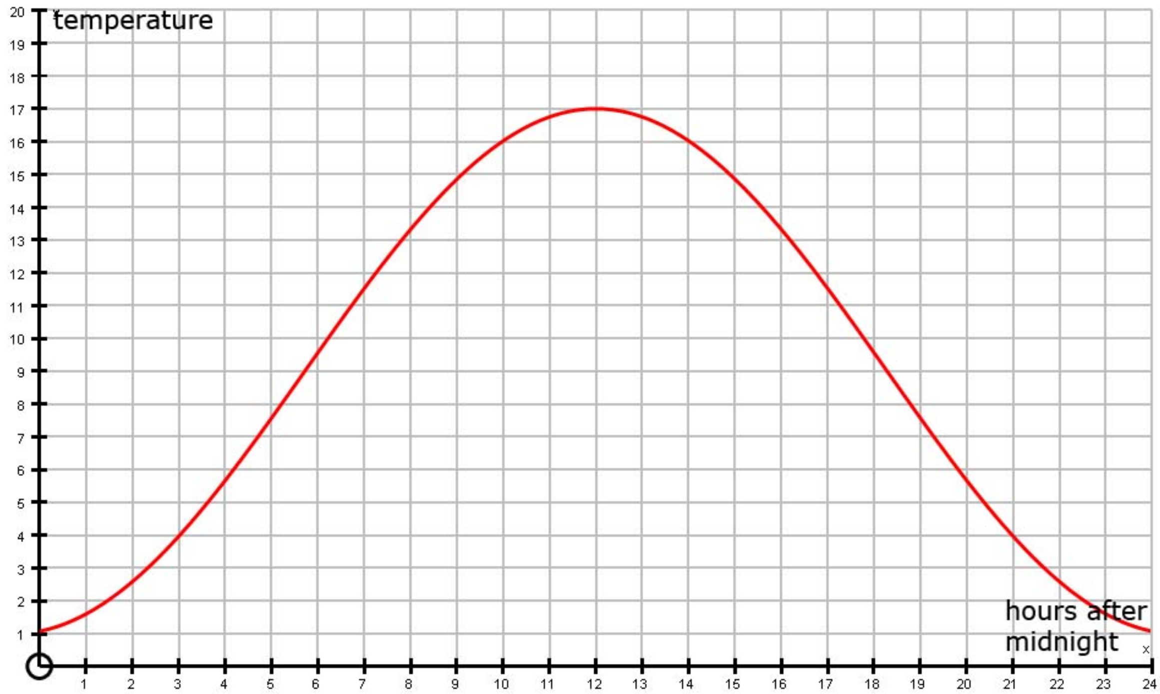
b)



c)



2 The average temperature in London for a day in March can be modeled by the function, below where *T* (on the *y*-axis) is the temperature and *x* is the hours after midnight.



a) Use the graph to find the temperature at 3AM.

b) Find the temperature at 2PM.

c) State the times when the the temperature is below 8^0.

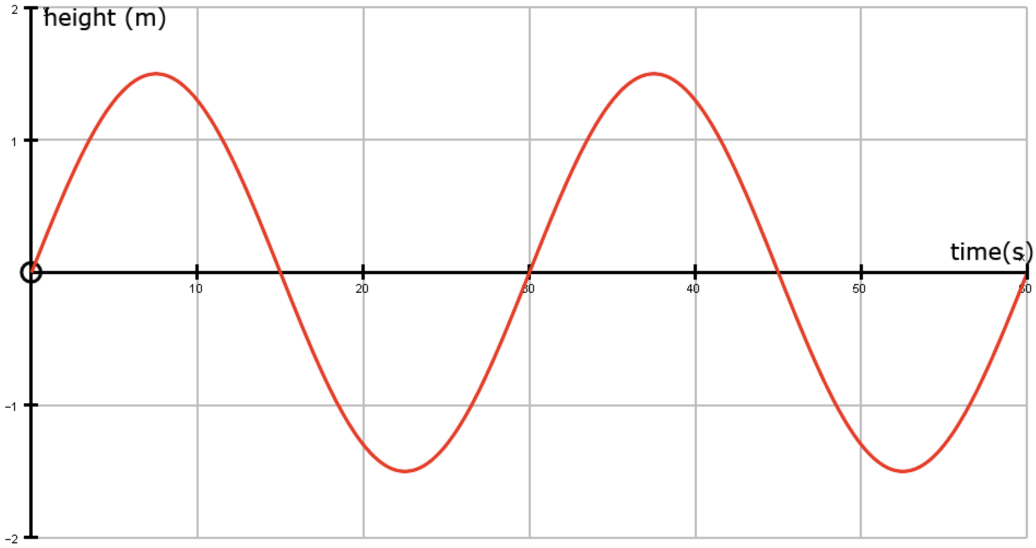
3 A sound wave follows the model $h = 4\cos(3x) + 1$, where x is time in seconds.

a) State the amplitude of the sound wave.

b) State the maximum and minimum values of the sound wave.

c) State the period of the sound wave.

4 The graph below shows the height of a buoy bobbing on a wave.

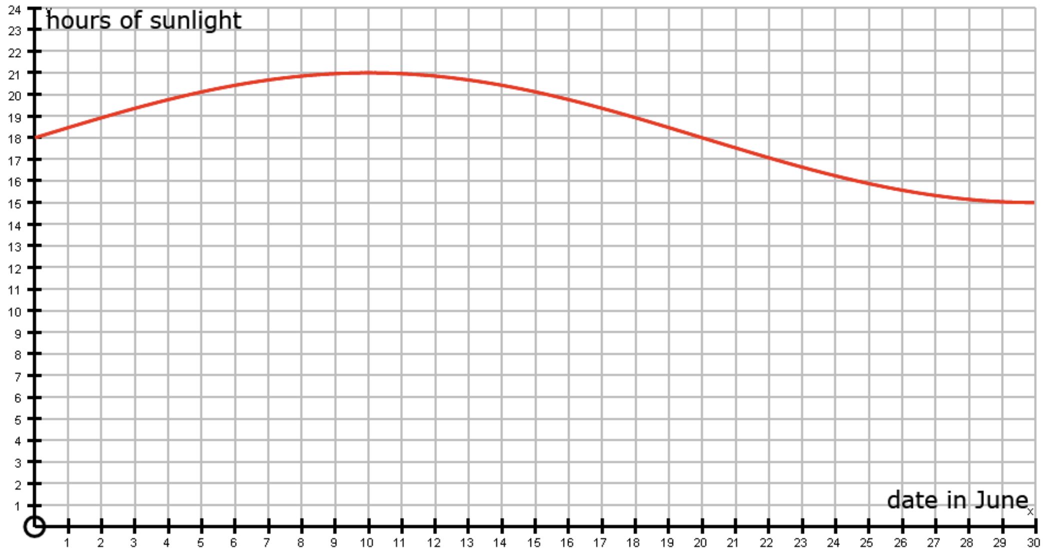


The height of the buoy is modeled by the function,

$h(x) = a \sin bx$.

Find the values of a and b.

5



The graph above shows the hours of daylight in a Scandinavian city during the month of June.

a) Find the date when the city has the most ours of daylight.

b) Find the number of hours of daylight on June 20th.

Trigonometric Relationships

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

1 *Non calculator.*
It is given that $\sin x = 0.5$ and the angle x is obtuse. Find each of the following, leaving your answer in exact form,

a) $\cos x$

b) $\tan x$

c) $\sin 2x$

d) $\cos 2x$

2 Solve the equation,

$3\sin^2\theta - 5\cos\theta = 1 \quad 0 \le \theta \le 2\pi$

3 Solve the equation,

$3\sin\theta = 5\cos\theta \quad 0 \le \theta \le 2\pi$

4 *Non calculator.*
It is given that $\cos 2x = 0.4$ and the angle $2x$ is reflex.
Find each of the following, leaving your answer in exact form,

a) $\sin x$

b) $\cos x$

c) $\sin 2x$

d) $\tan x$

e) $\tan 2x$

5 a) Sketch the graph of,

$$y=2\sin\theta+3\tan\theta-\left(\frac{\pi}{2}\right) \quad 0\leq\theta\leq2\pi$$

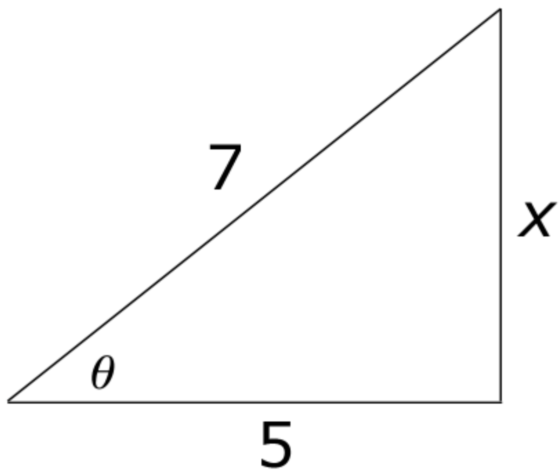
b) Solve the equation,

$$2\sin\theta+3\tan\theta-\left(\frac{\pi}{2}\right)=0 \quad 0\leq\theta\leq2\pi$$

6 By using trigonometric relationships solve the equation,

$$2sin\theta cos\theta+cos^2 2\theta-sin^2 2\theta+1=0 \quad 0\leq\theta\leq2\pi$$

7 *Non calculator.*
The diagram below shows a right-angled triangle. Use the triangle to answer each of a) to f) below giving your answers as exact numbers.



a) the missing side, *x*.

b) $sin\theta$

c) $cos\theta$

d) $tan\theta$

e) $sin2\theta$

f) $cos2\theta$

Double Angle Relationships

You need to be in radian mode.

Complete the table below, choose your own value for the last row. Give all your answers to 3 significant figures.

x	$\sin 2x$	$2\sin^2 x$	$2\sin x\cos x$	$\cos 2x$	$2\cos^2 x$	$\cos^2 x-\sin^2 x$
0.5						
0.8						
0.3						
-0.7						

What do you notice?

$\sin 2x$ = ...

$\cos 2x$ = ...

$\cos 2x$ = ... -1

$\cos 2x$ = 1-.................................