Topic 1 - Algebra (Exponentials and Logarithms)

Exponentials

1 Find the value of x in each of the following.

a) $2^{3x-4} = 128$

b) $4^{2x+1} = 2^4$

c) $27^{4x} = 9^{6x}$

d) $\frac{2^{3x}}{4} = 8 \times 2^{5x-9}$

2 a) Write 8×2^n as a power of 2.

b) Write $\frac{81}{3^{2n}}$ as a power of 3.

c) Simplify $\frac{2^{4x+2}}{2^{1-x}}$.

3 Given that $8^a 2^b = 1$ and $\frac{4^a}{2^{-b}} = 8$ find a and b.

4 In 1250 the population of England was 8 million. In the following years the Black Death struck and the population declined by 3% each year. Mathematicians have modeled the population by,

$P_n = P_0(0.97)^n$ where n is years after 1250.

a) Write down the value of P_0.

b) Use the model to find the population of England in 1270.

By 1300 the population had hit 4m and and started to grow steadily again by 4% per year.

c) Write down a model to show the population growth of England from 1300 onwards.

d) Hence, find the year in which England's population once again was 8 million.

5 A model for the number of bacteria in an experiment is given as,

$P_n = 500 + ae^n$ where n is hours after the start.

The initial value of the bacteria is 700.

a) Evaluate the value of *a*.

b) Find the number of bacteria after 2 hours.

c) Using a GDC find the number of bacteria after 10 hours.

d) Use a GDC to find the the value of *n* when the bacteria first reaches 1 million.

6 Isotopes, once used in science experiments, are decreasing in their value exponentially such that the value they can be modeled by,

$P_n = P_0 e^{-0.85t}$

where t is the time, in hours, since the experiment finished. The initial value of the isotope is known to be 12000.

a) Find the value of the isotope after 5 hours.

b) Use a graph to find the number of hours that will have passed before the value of the isotope has halved.

<u>Logarithms and Exponents</u>

1 Non-calculator

Given that $A = \ln 5$ and $B = \ln 2$, express in terms of A and B,

a) ln500

b) $\log_{16}\sqrt{5}$

2 Non-calculator

Solve the following simultaneous equations,

$2^a 8^b = 128$ and $\frac{4^a}{8^{4b}} = 32.$

3 Find the value of *x* correct to 2 decimal places.

a) $e^{\frac{1}{2}x-3} = 20.086$

b) $\ln(x+4) = 2.303$

4 A function is defined as $f(x) = \log_5(x-1) + 5.$

a) Write down the domain and range of *f(x)*.

b) Sketch a graph of *f(x)*, showing all intercepts with the axis, and the asymptotes.

c) Find $f^{-1}(x).$

5 A radioactive substance is decaying such that the weight at time *t* days from when the radiation occurred is given by the function,

$$w_t = 1000 + 2000e^{-0.05t}$$

a) Find the initial weight of the substance.

b) Find the value of the weight as the time approaches infinity.

c) The weight after 10 days.

d) Find the number of days before the substances' weight is halved.

Answers:

Exponentials

1 a) $x = \frac{11}{3}$

b) $x = \frac{1}{2}$

c) x any real number.

d) $x = 2$

2 a) 2^{3+n}

b) 3^{4-2n}

c) 2^{5x+1}

3 $a = -3, b = 9$

4 a) 8 million.

b) 4.35 million.

c) $P_n = 4(1.04)^n$

d) 1317

5 a) a=200

b) 1978

c) 4405793

d) n=8.5

6 a) 171.2

b) 0.82

Logarithms and Exponents

Logarithms and Exponents

1 a) $3A + 2B$

b) $\frac{A}{8B}$

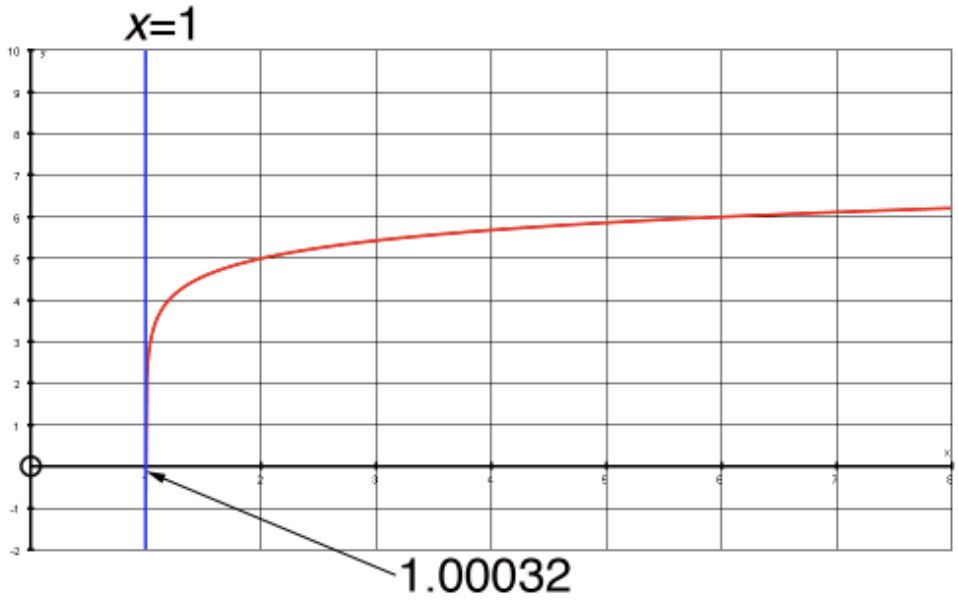
2 $a = \frac{11}{2}; b = \frac{1}{2}$

3 a) $x = 12$

b) $x = 6$

4 a) Domain $x>1$
Range can be any real number

b)



c) $f(x)^{-1} = 5^{x+5} + 1$

5 a) 3000

b) 1000

c) 2213

d) 27.7 (27th day)