

# IB Math SL

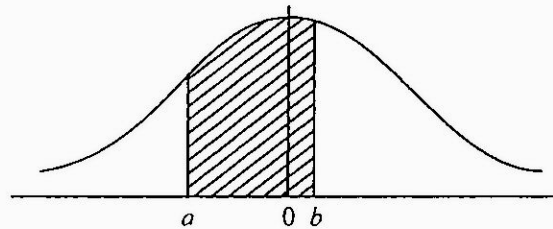
## Probability and Statistics Practice Test

Feb. 2008

*No calculator allowed. Use formula sheet*

1. Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds. **USE MM GRAPHING PAPER ON NEXT PAGE.**

- (a) The graph below is that of the **standard** normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.



- (i) Write down the value of  $a$  and of  $b$ .
- (ii) Calculate the probability that the reaction time of a person chosen at random is
- (a) greater than 0.70 seconds;
- (b) between 0.70 and 0.79 seconds.

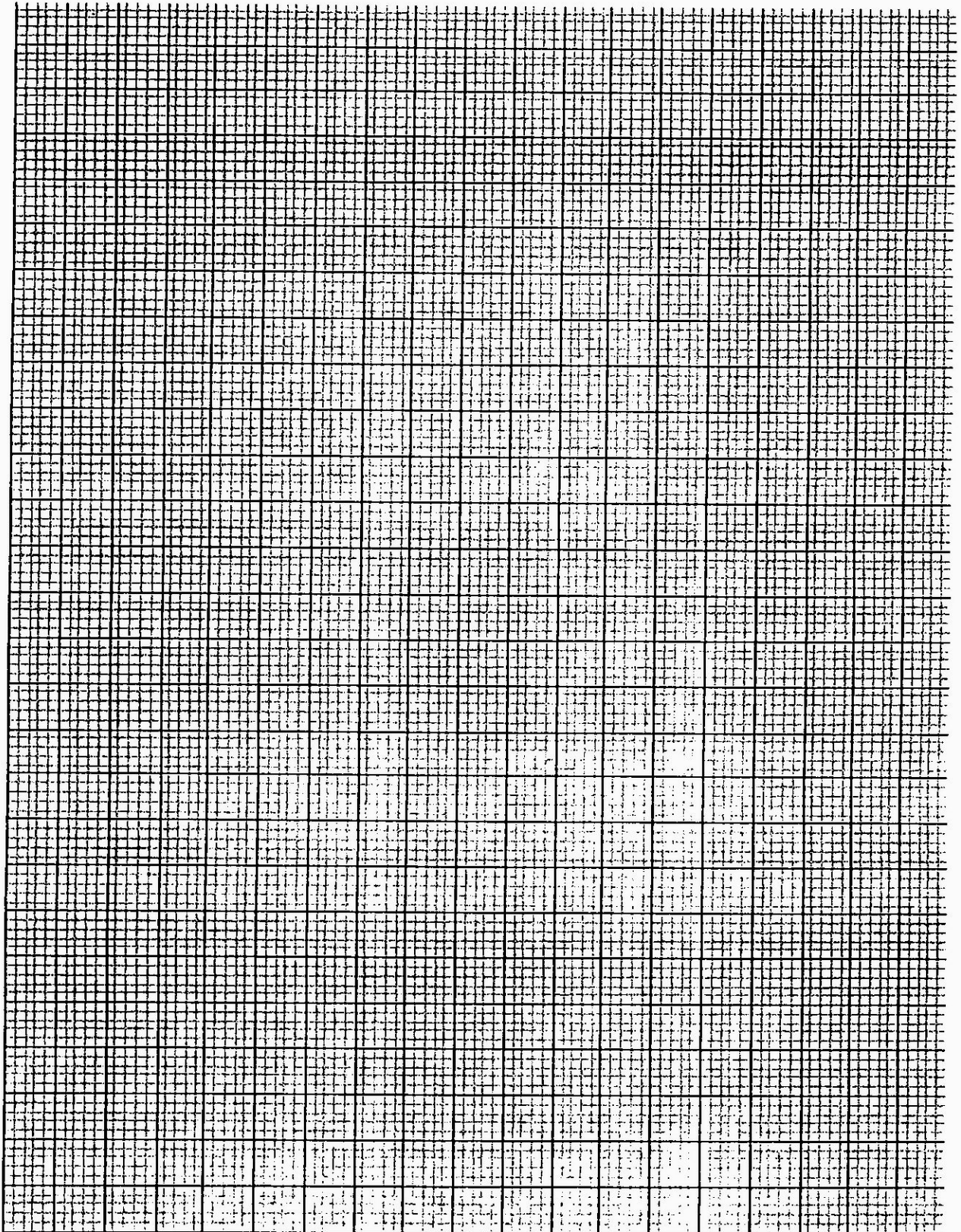
(6)

Three percent (3 %) of the population have a reaction time less than  $c$  seconds.

- (b) (i) Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3 %.
- (ii) Find  $c$ .

(4)

(Total 10 marks)



2. The weights of a certain species of bird are normally distributed with mean 0.8 kg and standard deviation 0.12 kg. Find the probability that the weight of a randomly chosen bird of the species lies between 0.74 kg and 0.95 kg.

*Working:*

*Answer:*

.....

(Total 6 marks)

3. A coin is biased so that when it is tossed the probability of obtaining heads is  $\frac{2}{3}$ . The coin is tossed 1800 times. Let  $X$  be the number of heads obtained. Find
- (a) the mean of  $X$ ;
- (b) the standard deviation of  $X$ .

*Working:*

*Answers:*

(a) .....

(b) .....

(Total 3 marks)

4. A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

- (a) Write down the expected number of faulty calculators in the sample.
- (b) Find the probability that three calculators are faulty.
- (c) Find the probability that more than one calculator is faulty.

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**(Total 6 marks)**

5. A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.

- (a) In eight such selections, what is the probability that a black disc is selected
  - (i) exactly once? (3)
  - (ii) at least once? (3)
- (b) The process of selecting and replacing is carried out 400 times.  
What is the expected number of black discs that would be drawn? (2)

**(Total 8 marks)**

6. For events  $A$  and  $B$ , the probabilities are  $P(A) = \frac{3}{11}$ ,  $P(B) = \frac{4}{11}$ .

Calculate the value of  $P(A \cap B)$  if

- (a)  $P(A \cup B) = \frac{6}{11}$ ;  
(b) events  $A$  and  $B$  are independent.

*Working:*

*Answers:*

- (a) .....  
(b) .....

**(Total 6 marks)**

7. A painter has 12 tins of paint. Seven tins are red and five tins are yellow. Two tins are chosen at random. Calculate the probability that both tins are the same colour.

*Working:*

*Answer:*

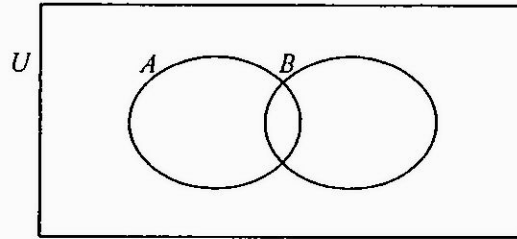
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**(Total 6 marks)**





10. The following Venn diagram shows a sample space  $U$  and events  $A$  and  $B$ .



$n(U) = 36$ ,  $n(A) = 11$ ,  $n(B) = 6$  and  $n(A \cup B)' = 21$ .

- (a) On the diagram, shade the region  $(A \cup B)'$ .
- (b) Find
- (i)  $n(A \cap B)$ ;
  - (ii)  $P(A \cap B)$ .
- (c) Explain why events  $A$  and  $B$  are not mutually exclusive.

*Working:*

*Answers:*

- (b) (i) .....
- (ii) .....
- (c) .....

**(Total 4 marks)**



11. The mean of the population  $x_1, x_2, \dots, x_{25}$  is  $m$ . Given that  $\sum_{i=1}^{25} x_i = 300$  and

$$\sum_{i=1}^{25} (x_i - m)^2 = 625, \text{ find}$$

- (a) the value of  $m$ ;
- (b) the standard deviation of the population.

<i>Working:</i>	<i>Answers:</i>
	(a) .....
	(b) .....

(Total 4 marks)

12. At a conference of 100 mathematicians there are 72 men and 28 women. The men have a mean height of 1.79 m and the women have a mean height of 1.62 m. Find the mean height of the 100 mathematicians.

<i>Working:</i>	<i>Answers:</i>
	.....

(Total 4 marks)

13. The table shows the scores of competitors in a competition.

Score	10	20	30	40	50
Number of competitors with this score	1	2	5	$k$	3

The mean score is 34. Find the value of  $k$ .

*Working:*

*Answers:*

.....

(Total 4 marks)

14. The number of hours of sleep of 21 students are shown in the frequency table below

Hours of sleep	Number of students
4	2
5	5
6	4
7	3
8	4
10	2
12	1

Find

- (a) the median;
- (b) the lower quartile;
- (c) the interquartile range.

<i>Working:</i>	<i>Answers:</i> (a) ..... (b) ..... (c) .....
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**(Total 6 marks)**

15. A teacher drives to school. She records the time taken on each of 20 randomly chosen days. She finds that

$$\sum_{i=1}^{20} x_i = 626 \text{ and } \sum_{i=1}^{20} x_i^2 = 19780.8, \text{ where } x_i \text{ denotes the time, in minutes, taken on the } i^{\text{th}} \text{ day.}$$

Calculate an unbiased estimate of

- (a) the mean time taken to drive to school;  
 (b) the variance of the time taken to drive to school.

<p><i>Working:</i></p>          	<p><i>Answers:</i></p> <p>(a) .....</p> <p>(b) .....</p>
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(Total 6 marks)

- 16 Let  $a, b, c$  and  $d$  be integers such that  $a < b, b < c$  and  $c = d$ .

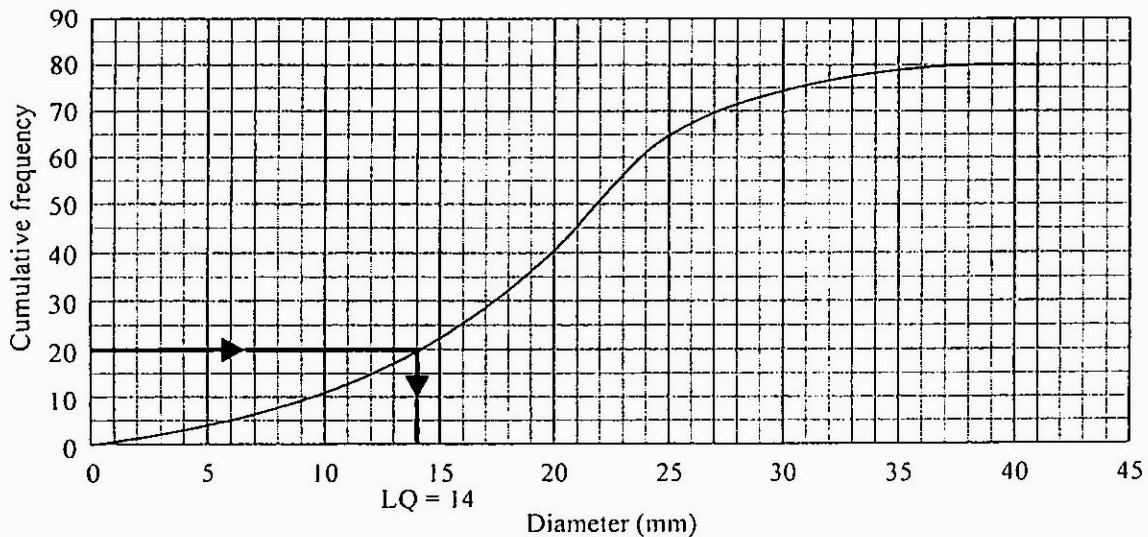
The mode of these four numbers is 11.  
 The range of these four numbers is 8.  
 The mean of these four numbers is 8.

Calculate the value of each of the integers  $a, b, c, d$ .

<p><i>Working:</i></p>          	<p><i>Answers:</i></p> <p><math>a = \dots, b = \dots</math></p> <p><math>c = \dots, d = \dots</math></p>
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(Total 6 marks)

17. A student measured the diameters of 80 snail shells. His results are shown in the following cumulative frequency graph. The lower quartile (LQ) is 14 mm and is marked clearly on the graph.



- (a) On the graph, mark clearly in the same way and write down the value of
- the median;
  - the upper quartile.
- (b) Write down the interquartile range.

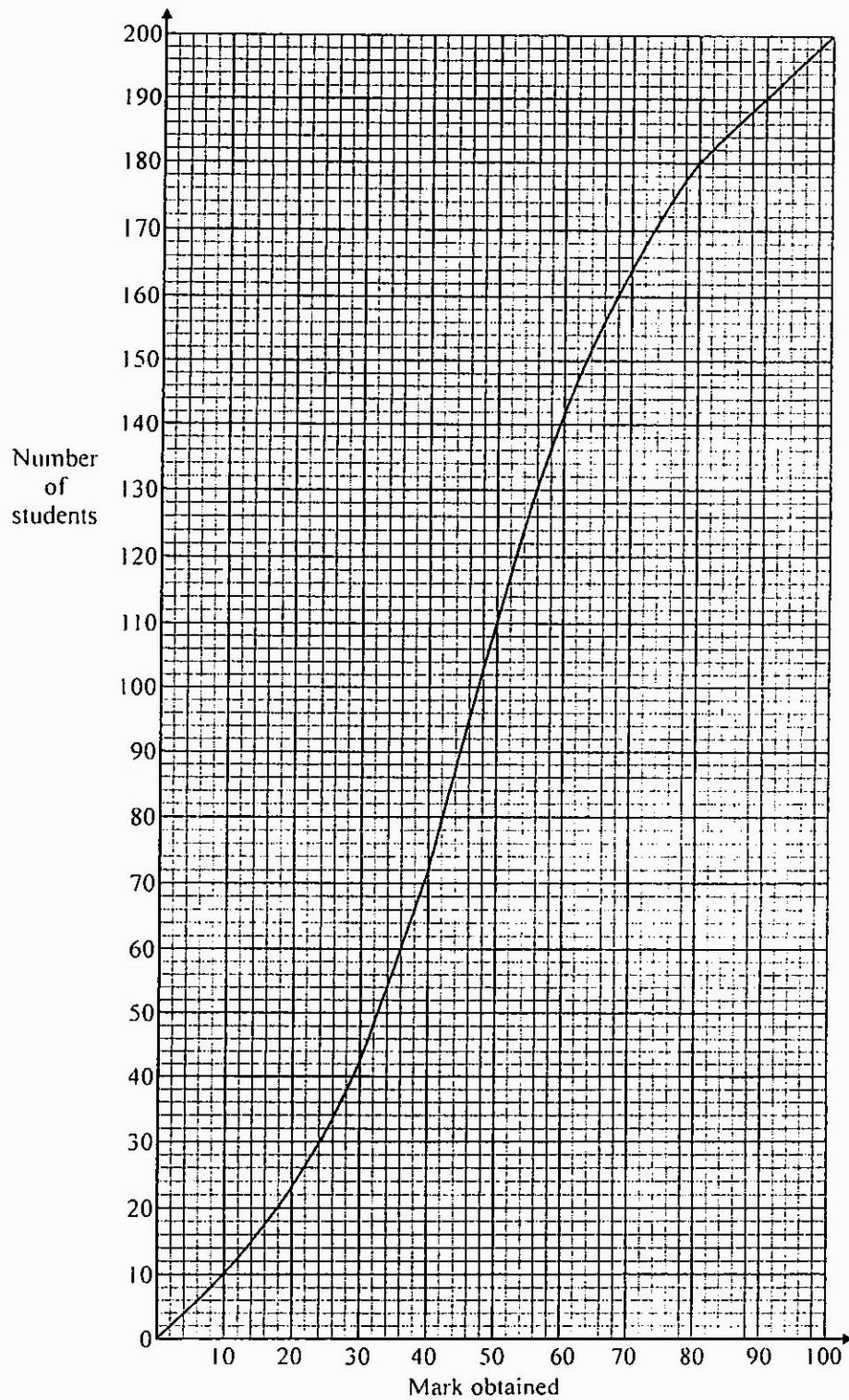
*Working:*

*Answer:*

(b) .....

(Total 6 marks)

18. The cumulative frequency curve below shows the marks obtained in an examination by a group of 200 students.



(a) Use the cumulative frequency curve to complete the frequency table below.

Mark ( $x$ )	$0 \leq x < 20$	$20 \leq x < 40$	$40 \leq x < 60$	$60 \leq x < 80$	$80 \leq x < 100$
Number of students	22				20

(b) Forty percent of the students fail. Find the pass mark.

*Working:*

*Answer:*

(b) .....

(Total 6 marks)

19. In a suburb of a large city, 100 houses were sold in a three-month period. The following **cumulative frequency table** shows the distribution of selling prices (in thousands of dollars).

Selling price $P$ (\$ 1000)	$P \leq 100$	$P \leq 200$	$P \leq 300$	$P \leq 400$	$P \leq 500$
Total number of houses	12	58	87	94	100

- (a) Represent this information on a cumulative frequency **curve**, using a scale of 1 cm to represent \$ 50000 on the horizontal axis and 1 cm to represent 5 houses on the vertical axis. (4)
- (b) Use your curve to find the interquartile range. (3)

The information above is represented in the following frequency distribution.

Selling price $P$ (\$ 1000)	$0 < P \leq 100$	$100 < P \leq 200$	$200 < P \leq 300$	$300 < P \leq 400$	$400 < P \leq 500$
Number of houses	12	46	29	$a$	$b$

- (c) Find the value of  $a$  and of  $b$ . (2)
- (d) Use mid-interval values to calculate an estimate for the mean selling price. (2)
- (e) Houses which sell for more than \$ 350 000 are described as *De Luxe*.
- (i) Use your graph to estimate the number of *De Luxe* houses sold. Give your answer to the nearest integer.
- (ii) Two *De Luxe* houses are selected at random. Find the probability that **both** have a selling price of more than \$ 400 000. (4)

(Total 15 marks)