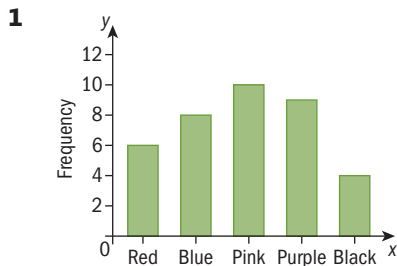


8

Descriptive statistics

Answers

Skills check



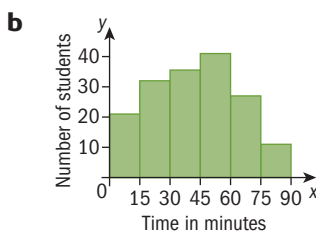
- 2 a Mean = $\frac{4+7+7+8+6}{5} = \frac{32}{5} = 6.4$
 b The number that occurs most often is 8
 c i Arrange the data in order of size. 2, 4, 4, 6, 7, 8, 11. The median is the middle member, 6
 ii 5, 7, 9, 11, 13, 15. The middle member is in between 9 and 11. $\frac{1}{2}(9 + 11) = 10$
 iii 6, 6, 11, 11, 14, 17. The middle member is between the two number elevens. 11

Exercise 8A

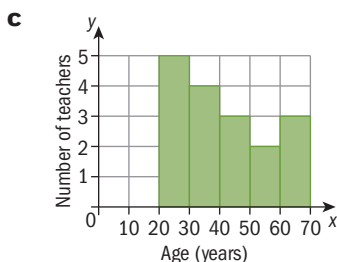
- 1 a Discrete. b Continuous.
 c Continuous. d Discrete
 2 Discrete

Exercise 8B

- 1 a Continuous



- 2 a Continuous
 b 17



- 3 a Continuous

b

Weight (kg)	$1 \leq w < 2$	$2 \leq w < 3$	$3 \leq w < 4$	$4 \leq w < 5$
Number of chickens	8	24	50	14

- c $8 + 24 + 50 + 14 = 96$

- 4 a Continuous

b

Time	f
$5 \leq t < 10$	1
$10 \leq t < 15$	2
$15 \leq t < 20$	4
$20 \leq t < 25$	4
$25 \leq t < 30$	2
$30 \leq t < 35$	2
$35 \leq t < 40$	1
$40 \leq t < 45$	1

- c 5 mins

Exercise 8C

- 1 a 18 b 9 c 18 and 24
 d 0 e $\frac{1}{2}$ and 2.
 2 a 1 b $170 \leq t < 180$

Exercise 8D

1 Mean = $\frac{\sum x}{n} = \frac{66+57+71+69+58+54}{6} = \frac{375}{6} = 62.5 \text{ kmh}^{-1}$

2 Mean = $\frac{\sum x}{n} = \frac{1.79+1.61+1.96+2.08}{4} = \frac{7.44}{4} = \1.86

- 3 a Discrete

b

Calls per day (x)	f	fx
2	3	6
3	2	6
4	5	20
5	3	15
6	4	24
7	6	42
8	3	24
9	4	36
Totals	30	173

Mean = $\frac{\sum fx}{\sum f} = \frac{173}{30} = 5.7\bar{6}$ calls per day.

- 4 a Continuous b $90 \leq m < 120$

c

Minutes (m)	f	Midpoint(m)	fm
$0 \leq m < 30$	12	15	180
$30 \leq m < 60$	16	45	720
$60 \leq m < 90$	20	75	1500
$90 \leq m < 120$	36	105	3780
$120 \leq m < 150$	16	135	2160
Totals	100		8340

Mean = $\frac{\sum fm}{\sum f} = \frac{8340}{100} = 83.4$ minutes per day.

- 5 Let x be Kelly's score on the fifth test. To average 84

$$84 = \frac{95 + 82 + 76 + 88 + x}{5}$$

$$84 \times 5 = 341 + x$$

$$420 = 341 + x$$

$$x = 420 - 341$$

$$x = 79$$

Kelly must score 79 on the next test.

- 6 Total mass of 11 players = $11 \times 80.3 = 883.3$ kg

$$81.2 = \frac{883.3 + x}{12}$$

$$81.2 \times 12 = 883.3 + x$$

$$974.4 = 883.3 + x$$

$$x = 974.4 - 883.3$$

$$x = 91.1 \text{ kg}$$

The new player has mass 91.1 kg

- 7 Let x be the distance they travel on sixth day.

$$250 = \frac{220 + 300 + 210 + 275 + 240 + x}{6}$$

$$250 \times 6 = 1245 + x$$

$$1500 = 1245 + x$$

$$x = 1500 - 1245$$

$$x = 255$$

They must travel 255 km on the last day.

- 8 Total number of shots = $8 \times 71 = 568$.

- 9 After 8 matches, the total points scored is

$$8 \times 27 = 216$$

After 11 matches the total points scored is

$$11 \times 29 = 319$$

$$319 - 216 = 103 \text{ points scored in the last 3 matches}$$

- 10 Billy's total = $12 \times 310 = \$3720$

$$\text{Jean's total} = 13 \times 320 = \$4160$$

$$\text{Billy's total} + \text{Jean's total} = \$7880$$

$$\text{Mean} = \frac{7880}{25} = \$315.20$$

Exercise 8E

- 1 a Arrange in order of size 2, 2, 3, 3, 4, 4, 5, 6, 7

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}} = 4$$

- b 2, 2, 3, 5, 5, 7, 8

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{7+1}{2}\right)^{\text{th}} = 4^{\text{th}} = 5$$

- c 0, 2, 3, 3, 4, 6, 7, 9

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{8+1}{2}\right)^{\text{th}} = 4.5^{\text{th}} = 3.5$$

- d 0, 1, 1.5, 2, 4, 4, 5, 8, 8.4, 9, 12

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{11+1}{2}\right)^{\text{th}} = 6^{\text{th}} = 4$$

- e 1, 2, 4, 5, 7, 9, 12, 20

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{8+1}{2}\right)^{\text{th}} = 4.5^{\text{th}} = 6$$

- 2 Number of CDs = $3+2+2+1+3+5+3 = 19$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{19+1}{2}\right)^{\text{th}} = 10^{\text{th}} = 11$$

- 3 Mean = 0, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 10.

$$\text{Mean} = \frac{\text{sum of scores}}{\text{Number of students}} = \frac{168}{32} = 5.25$$

$$\text{Mode} = 7$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{32+1}{2}\right)^{\text{th}} = 16.5^{\text{th}} = 5.5$$

Investigation – measures of central tendency

	Data	Mean	Mode	Median
Data Set	6, 7, 8, 10, 12, 14, 14, 15, 16, 20	12.2	14	13
Add four to each data set	10, 11, 12, 14, 16, 18, 18, 19, 20, 24	16.2	18	17
Multiply the original data set by 2	12, 14, 16, 20, 24, 28, 28, 30, 32, 40	24.4	28	26

- a If you add 4 to each data value, you will add 4 to the mean, mode and median.
- b If you multiply each data value by 2, you will multiply the mean, mode and median by 2.

Exercise 8F

- 1 a Range = largest – smallest = $125 - 30 = 95$ cm

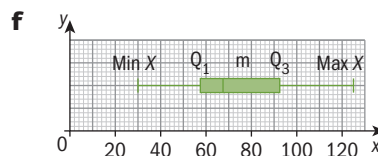
- b In ascending order, depths are 30, 45, 55, 60, 65, 65, 70, 75, 75, 110, 120, 125

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{12+1}{2}\right)^{\text{th}} = 6.5^{\text{th}} = 67.5$$

$$\text{c } Q_1 = \frac{1}{4}(n+1)^{\text{th}} = \frac{1}{4}(12+1)^{\text{th}} = 3.25^{\text{th}} = 57.5$$

$$\text{d } Q_3 = \frac{3}{4}(n+1)^{\text{th}} = \frac{3}{4}(12+1)^{\text{th}} = 9.75^{\text{th}} = 92.5$$

$$\text{e } IQR = Q_3 - Q_1 = 92.5 - 57.5 = 35$$



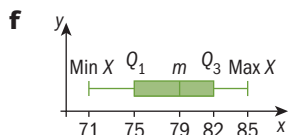
- 2 a Range = largest – smallest = $85 - 71 = 14$

$$\text{b } \text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{11+1}{2}\right)^{\text{th}} = 6^{\text{th}} = 79$$

$$\text{c } Q_1 = \frac{1}{4}(n+1)^{\text{th}} = \frac{1}{4}(11+1)^{\text{th}} = 3^{\text{rd}} = 75$$

$$\text{d } Q_3 = \frac{3}{4}(n+1)^{\text{th}} = \frac{3}{4}(11+1)^{\text{th}} = 9^{\text{th}} = 82$$

$$\text{e } IQR = Q_3 - Q_1 = 82 - 75 = 7$$



3 a Range = largest-smallest = $29 - 10 = 19$

b Median = $\left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{11+1}{2}\right)^{\text{th}} = 6^{\text{th}} = 21$

c $Q_1 = \frac{1}{4}(n+1)^{\text{th}} = \frac{1}{4}(11+1)^{\text{th}} = 3^{\text{rd}} = 12$

d $Q_3 = \frac{3}{4}(n+1)^{\text{th}} = \frac{3}{4}(11+1)^{\text{th}} = 9^{\text{th}} = 27$

e $IQR = Q_3 - Q_1 = 27 - 12 = 15$

4 a Range = $11 - 6 = 5$ **b** 8 **c** 7

d 10 **e** $IQR = 10 - 7 = 3$

5 Histogram **i**.

Shows range of $10 - 1 = 9$.

Shows sample size of $6 + 6 + 6 + 6 + 6 = 30$

Median = $\left(\frac{30+1}{2}\right)^{\text{th}} = 15.5^{\text{th}}$ which lies in 5–6 category

$Q_1 = \frac{1}{4}(30+1)^{\text{th}} = 7.75^{\text{th}}$ which lies in 3–4 category

$Q_2 = \frac{3}{4}(30+1)^{\text{th}} = 23.25^{\text{th}}$ which lies in 7–8 category

This information is shown by box plot **c**.

Histogram **ii**.

Shows range $10 - 1 = 9$.

Shows sample size of $4.5 + 7 + 8 + 4.5 + 3.5 = 27.5$.

Median = $\left(\frac{27.5+1}{2}\right)^{\text{th}} = 14.25^{\text{th}}$ which lies in 5–6 category.

$Q_1 = \frac{1}{4}(27.5+1)^{\text{th}} = 7.125^{\text{th}}$ lies in 3–4 category

$Q_2 = \frac{3}{4}(27.5+1)^{\text{th}} = 20.625^{\text{th}}$ lies in 7–8 category

This is shown in box plot **b**.

Histogram **iii**.

Shows range $10 - 1 = 9$.

Sample size: $7.5 + 2.5 + 5.5 + 3.5 + 8 = 27$

Median = $\left(\frac{27+1}{2}\right)^{\text{th}} = 14^{\text{th}}$ lies in 5–6 category

$Q_1 = \frac{1}{4}(27+1)^{\text{th}} = 7^{\text{th}}$ lies in 1–2 category

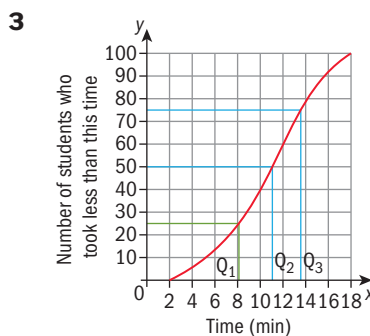
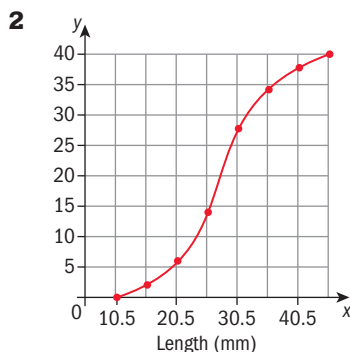
$Q_3 = \frac{3}{4}(27+1)^{\text{th}} = 21^{\text{th}}$ lies in 9–10 category

This is shown in box plot **a**.

Exercise 8G

1 a 74 cm. **b** $(77 - 71)$ cm = 6 cm.

c Half of the boxers have a reach of 71 cm to 77 cm.



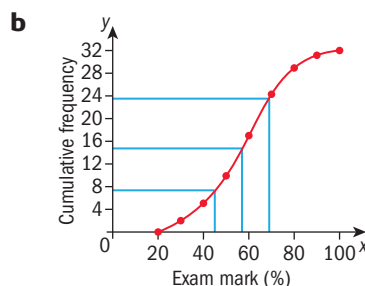
i 11 mins

ii $(13.6 - 8.2)$ mins = 5.4 mins.

b $p = 32, q = 8$.

4 a

Marks	<i>f</i>	CF
$20 \leq m < 30$	2	2
$30 \leq m < 40$	3	5
$40 \leq m < 50$	5	10
$50 \leq m < 60$	7	17
$60 \leq m < 70$	6	23
$70 \leq m < 80$	4	27
$80 \leq m < 90$	2	29
$90 \leq m < 100$	1	30



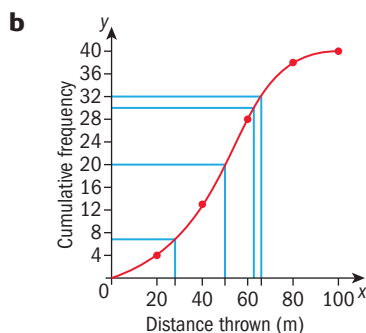
c i Median $\approx 57\%$

ii Lower quartile $\approx 45\%$
Upper quartile $\approx 69\%$

iii Interquartile range $\approx 69\% - 45\% = 24\%$

5 a

Distance (<i>m</i>)	<i>f</i>	CF
$0 \leq m < 20$	4	4
$20 \leq m < 40$	9	13
$40 \leq m < 60$	15	28
$60 \leq m < 80$	10	38
$80 \leq m < 100$	2	40

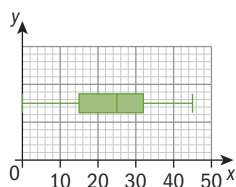


- c** 20% of 40 students = 8.
 Only top 8 students will made final.
 $40 - 8 = 32$.
 We draw a line across from 32 on y-axis, and down to see the required distance.
 Qualifying distance ≈ 66 m.

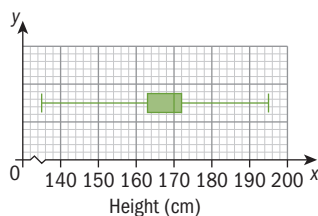
d Interquartile range $\approx 63 - 35 = 28$

e Median ≈ 50 m

- 6 a** 25 mins **b** 16 mins **c** 37 mins



- 7 a** 170 cm
b 50 flowers between 135 cm and 163 cm
c 20 flowers. 178 cm
d 100.



Exercise 8H

- 1 a** mean = 18, variance = 129.6 Standard deviation = 11.4
b mean = 40, variance = 200 standard deviation = 14.1
- 2 a** Variance = 78.5 standard deviation = 8.86
b Variance = 80.18 standard deviation = 8.95
c Variance = 449 Standard deviation = 21.2
- 3** 1.32
- 4** Mean = 2.5 Standard deviation = 1.24
- 5** Mean = 26.2, standard deviation = 14.9.
- 6 a** Discrete **b** 2.73 **c** 1.34 **d** 23
- 7** Mean = 42.4 standard deviation = 21.6
- 8 a** 51 **b** 69.5 **c i** 21.8 **ii** None

Investigation - the effect of adding or multiplying the data set of a standard deviation

- a** 2.47
b The mean has had 100 added to it.
c 2.47
d The standard deviation remains the same. This is because the standard deviation only measures the spread of the numbers, and that remains constant if the same number is added to each item in the list.
e The mean is doubled.
f 4.94
g The variance will be multiplied by 4 because the variance is the standard deviation squared.

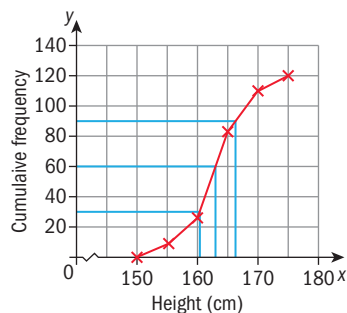


Review exercise

- 1 a** Mode = 3 as 3 appears the most in the list.
b First write the numbers in ascending order: 1, 2, 3, 3, 5, 6, 7, 8, 10
 Median = $\left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}} = 5$.
c Mean = $\frac{1+2+3+3+5+6+7+8+10}{9} = 5$
d Range = $10 - 1 = 9$.
- 2 a**
- | | | | | | | | | | |
|----------|---|----|----|----|----|---|---|---|----|
| Pets (p) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| f | 3 | 9 | 10 | 2 | 3 | 1 | 1 | 0 | 1 |
| fp | 6 | 27 | 40 | 10 | 18 | 7 | 8 | 0 | 10 |
- Mean = $\frac{\sum fp}{\sum f} = \frac{126}{30} = 4.2$.
- b** Median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ value = $\left(\frac{30+1}{2}\right)^{\text{th}} = 15.5^{\text{th}} = 4$
c Mode = 4.
- 3** Mean = 27.5 yrs, standard deviation = 0.4 yrs.
- 4 a** Median = 52
b Range $60 - 46 = 14$
c Inter Quartile range = $57 - 49 = 8$.
- 5 a** $\frac{46 + 92 + 4x}{6} = 71$.
 $4x = (6 \times 71) - 138$
 $x = \frac{288}{4} = 72$.
 So total = $46 + 92 + 4(72) = 426$
b $x = 72$. (from part a).
c New mean decreased by 6 also.
 New mean is $71 - 9 = 62$

6 a

Height	f	Σf
$150 \leq h < 155$	4	4
$155 \leq h < 160$	22	26
$160 \leq h < 165$	56	82
$165 \leq h < 170$	32	114
$170 \leq h < 175$	5	119



- b** Median ≈ 163 ,
c IQR ≈ 6 .
- 7 a** $26 + 10 + 20 + k + 29 + 11 = 100$
 $k = 100 - 96$
 $k = 4$
- b i** Median $= \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{100+1}{2}\right) = 50.5^{\text{th}} = 3$
ii $Q_1 = \frac{1}{4}(n+1)^{\text{th}} = \frac{1}{4}(100+1)^{\text{th}} = 25.25^{\text{th}} = 1$
 $Q_3 = \frac{3}{4}(n+1)^{\text{th}} = \frac{3}{4}(100+1)^{\text{th}} = 75.75^{\text{th}} = 5$
 Interquartile range $= 5 - 1 = 4$.
- 8** Total readings $= 6 + 3 + 5 + 8 + 6 + 2 = 30$
 Median $= \left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{30+1}{2}\right) = 15.5^{\text{th}} = 50$

Note: 50 is the midpoint of $42.5 \leq t \leq 57.5$, in which the 15.5^{th} value lies.

$$Q_1 = \frac{1}{4}(n+1)^{\text{th}} = \frac{1}{4}(31) = 7.75 = 35$$

$$Q_2 = \frac{3}{4}(n+1)^{\text{th}} = \frac{3}{4}(31) = 23.25 = 80$$

$$\text{IQR} = 80 - 35 = 45.$$



Review exercise

- 1** Median = 20 IQR = 14.
- 2 a** 6.48
b 1.31
- 3 a** 6
b 6
c 5.92
- 4 a** Mean = 2.57, median = 2, mode = 1 standard deviation = 1.68 and variance = 2.82.
b Range = 6, lower quartile = 1 and the interquartile range = 3.
- 5 a** $160 \leq \text{Height} < 170$
- b**
- | Height | f |
|--------------------------------|-----|
| $140 \leq \text{Height} < 150$ | 15 |
| $150 \leq \text{Height} < 160$ | 55 |
| $160 \leq \text{Height} < 170$ | 90 |
| $170 \leq \text{Height} < 180$ | 45 |
| $180 \leq \text{Height} < 190$ | 5 |
- 6 a i** $p = 65$
ii $q = 34$
b median = 18
c mean = 17.7