6 Patterns, sequences and series

Answers

Skills check

1 a $3x - 5 = 5x + 7$
$-2x = 12$
$x = -6$

b $p(2 - p) = -15$
$2p - p^2 = -15$
$p^2 - 2p - 15 = 0$
$(p + 3)(p - 5) = 0$
$p + 3 = 0, p - 5 = 0$
$p = -3, p = 5$

c $2^n + 9 = 41$
$2^n = 32 = 2^5$
$n = 5$

2 a $6m + 8k = 30$
$8k = 30 - 6m$
$k = \frac{30 - 6m}{8}$
$k = \frac{15 - 3m}{4}$

b $2pk - 5 = 3$
$2pk = 8$
$k = \frac{8}{2p}$
$k = \frac{4}{p}$

3 a $T = 2x(x + 3y)$
$T = 2(3)(3 + 3(5))$
$T = 6(18)$
$T = 108$

b $T = 2\,(4.7)(4.7 + 3(-2))$
$T = 9.4\,(-1.3)$
$T = -12.22$

4 a $m = 2^x - y^3$
$m = 2^5 - 3^3$
$m = 32 - 27$
$m = 5$

b $m = 2^3 - (-2)^3$
$m = 8 - (-8)$
$m = 16$

c $m = 2^{-5} - \left(\frac{1}{2}\right)^3$
$m = \frac{1}{32} - \frac{1}{8}$
$m = -\frac{3}{32}$

Investigation – saving money

Week Number	Weekly Savings	Total Savings
1	20	20
2	25	45
3	30	75
4	35	110
5	40	150
6	45	195
7	50	245
8	55	300

Joel saves \$65 in the 10th week, \$100 in the 17th week
He will save \$7670 in the first year.
It will take 17 weeks to save over \$1000.
$M = 20 + 5\,(n - 1)$
$T = \frac{5n^2 + 35n}{2}$

Exercise 6A

1 a 19, 23, 27 *(add 4 to the previous term)*

b 16, 32, 64 *(multiply previous term by 2)*

c 18, 24, 31 *(add 1, add 2, add 3, add 4, and so on...)*

d 80, −160, 320 *(multiply previous term by −2)*

e $\frac{9}{14}, \frac{11}{17}, \frac{13}{20}$ *(numerator increases by 2, denominator increases by 3)*

f 6.01234, 6.012345, 6.0123456 (The decimal places are consecutive integers).

2 a $u_1 = 10$, $u_2 = 3(10) = 30$, $u_3 = 3(30) = 90$, $u_4 = 3(90) = 270$

b $u_1 = 3$, $u_2 = 2(3) + 1 = 7$, $u_3 = 2(7) + 1 = 15$, $u_4 = 2(15) + 1 = 31$

c $u_1 = \frac{3}{4}$, $u_2 = \frac{2}{3}\left(\frac{3}{4}\right) = \frac{1}{2}$, $u_3 = \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3}$, $u_4 = \frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9}$

d $u_1 = x$, $u_2 = (x)^2 = x^2$, $u_3 = (x^2)^2 = x^4$, $u_4 = (x^4)^2 = x^8$

3 a $u_1 = 2$ and $u_{n+1} = u_n + 2$ (since each term is found by adding 2 to the previous term)

b $u_1 = 1$ and $u_{n+1} = 3u_n$ (since each term is found by multiplying the previous term by 3).

c $u_1 = 64$ and $u_{n+1} = \frac{u_n}{2}$ (since each term is found by multiplying the previous term by $\frac{1}{2}$).

d $u_1 = 7$ and $u_{n+1} = u_{n+5}$ (since each term is found by adding 5 to the previous term).

4 **a** $u_n = 3^n$. $u_1 = 3^1 = 3$, $u_2 = 3^2 = 9$, $u_3 = 3^3 = 27$, $u_4 = 3^4 = 81$

b $u_n = -6n + 3$. $u_1 = -6(1) + 3 = -3$, $u_2 = -6(2) + 3 = -9$, $u_3 = -6(3) + 3 = -15$ $u_4 = -6(4) + 3 = -21$

c $u_n = 2^{n-1}$. $u_1 = 2^0 = 1$, $u_2 = 2^1 = 2$, $u_3 = 2^2 = 4$, $u_4 = 2^3 = 8$.

d $u_n = n^n$. $u_1 = 1^1 = 1$, $u_2 = 2^2 = 4$, $u_3 = 3^3 = 27$, $u_4 = 4^4 = 256$.

5 **a**

Term number	1	2	3	4	…	n
	$\downarrow \times 2$	$\downarrow \times 2$	$\downarrow \times 2$	$\downarrow \times 2$		$\downarrow \times 2$
Term	2	4	6	8	…	$2n$

To get each term, we multiply the term number by 2. So $u_n = 2n$

b

Term number	1	2	3	4	…	n
	$\downarrow 3^0$	$\downarrow 3^1$	$\downarrow 3^2$	$\downarrow 3^3$		$\downarrow 3^{n-1}$
Term	1	3	9	27		3^{n-1}

To get each term u_n, we raise 3 to the power of $n - 1$. So $u_n = 3^{n-1}$

c

Term number	1	2	3	4	…	n
	$\downarrow 2^{7-1}$	$\downarrow 2^{7-2}$	$\downarrow 2^{7-3}$	$\downarrow 2^{7-4}$		$\downarrow 2^{7-n}$
Term	64	32	16	8		2^{7-n}

To get each term, u_n we raise 2 to the power $(7 - n)$. Thus, $u_n = 2^{7-n}$

d

Term number	1	2	3	4	n
	$\downarrow (5\times1)+2$	$\downarrow (5\times2)+2$	$\downarrow (5\times3)+2$	$\downarrow (5\times4)+2$	$\downarrow (5\times3)+2$
Term	7	12	17	22	$5n+2$

To get each term, u_n, we multiply n by 5 and add 2.

e

Term number	1	2	3	4	n
	$\downarrow \frac{1}{1+1}$	$\downarrow \frac{2}{2+1}$	$\downarrow \frac{3}{3+1}$	$\downarrow \frac{4}{4+1}$	$\downarrow \frac{n}{n+1}$
Term	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{n}{n+1}$

To get each term, u_n, we divide n by $n + 1$. So $u_n = \frac{n}{n+1}$.

f

Term number	1	2	3	4	n
	$\downarrow 1\times x$	$\downarrow 2\times x$	$\downarrow 3\times x$	$\downarrow 4\times x$	$\downarrow n\times x$
Term	x	$2x$	$3x$	$4x$	$\downarrow n\times x$

To get each term, u_n, we multiply x by n. So $u_n = nx$.

6 **a** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 the 15th term is 610

b $u_1 = 1$, $u_2 = 1$, and $u_{n+1} = u_n + u_{n-1}$

Exercise 6B

1 **a** **i** $u_n = u_1 + (n - 1)d$. $u_1 = 3$ and $d = 3$,
$u_{15} = 3 + (15 - 1)3$
$u_{15} = 3 + (14)3$
$u_{15} = 45$

ii $u_n = 3 + (n - 1)3$
$u_n = 3n$

b **i** $u_n = u_1 + (n - 1)d$. $u_1 = 25$ and $d = 15$,
$u_{15} = 25 + (15 - 1)15$
$u_{15} = 25 + (14)15$
$u_{15} = 235$

ii $u_n = 25 + (n - 1)15$
$u_n = 15n + 10$

c **i** $u_n = u_1 + (n - 1)d$. $u_1 = 36$ and $d = 5$,
$u_{15} = 36 + (15 - 1)5$
$u_{15} = 36 + (14)5$
$u_{15} = 106$

ii $u_n = 36 + (n - 1)5$
$u_n = 5n + 31$

d **i** $u_n = u_1 + (n - 1)d$. $u_1 = 100$ and $d = -13$,
$u_{15} = 100 + (15 - 1)(-13)$
$u_{15} = 100 + (14)(-13)$
$u_{15} = -82$

ii $u_n = 100 + (n - 1)(-13)$
$u_n = 113 - 13n$

e **i** $u_n = u_1 + (n - 1)d$. $u_1 = 5.6$ and $d = 0.6$,
$u_{15} = 5.6 + (15 - 1)(0.6)$
$u_{15} = 5.6 + (14)(0.6)$
$u_{15} = 14$

ii $u_n = 5.6 + (n - 1)(0.6)$
$u_n = 0.6n + 5$

f **i** $u_n = u_1 + (n - 1)d$. $u_1 = x$ and $d = a$,
$u_{15} = x + (15 - 1)(a)$
$u_{15} = x + 14a$

ii $u_n = x + (n - 1)(a)$
$u_n = x + an - a$

2 **a** $u_1 + (n - 1)d = u_n$
$5 + (n - 1)5 = 255$
$5n = 255$
$n = 51$

b $u_1 + (n - 1)d = u_n$
$4.8 + (n - 1)(0.2) = 38.4$
$0.2n + 4.6 = 38.4$
$0.2n = 38$
$n = 169$

c $u_1 + (n - 1)d = u_n$
$\frac{1}{2} + (n-1)\left(\frac{3}{8}\right) = 14$
$\frac{3}{8}n + \frac{1}{8} = 14$
$\frac{3}{8}n = \frac{111}{8}$
$n = 37$

d $u_1 + (n-1)d = u_n$
$250 + (n-1)(-29) = -156$
$-29n + 279 = -156$
$-29n = -435$
$n = 15$

e $u_1 + (n-1)d = u_n$
$2m + (n-1)(3m) = 80m$
$3mn - m = 80m$
$3mn = 81m$
$n = 27$

f $u_1 + (n-1)d = u_n$
$x + (n-1)(2x+3) = 19x + 27$
$n(2x+3) - x - 3 = 19x + 27$
$n(2x+3) = 20x + 30 = 10(2x+3)$
$n = 10$

Exercise 6C

1 $u_{15} = 19 + (15-1)d = 31.6$
$19 + 14d = 31.6$
$14d = 12.6$
$d = 0.9$

2 $u_{10} = u_1 + (10-1)d = 37$
$u_1 + 9d = 37$ *(call this equation #1)*
$u_{21} = u_1 + (21-1)d = 4$
$u_1 + 20d = 4$ *(call this equation #2)*
(solve using simultaneous equation solver on GDC)
$u_1 = 64,\ d = -3$

3 $3 + 2d = 8$
$d = 2.5$
$x = 3 + 2.5$
$x = 5.5$

4 since $u_1 + d = u_2$, $m + d = 13$ *(call this equation #1)*
since $u_2 + d = u_3$, $13 + d = 3m - 6$
$3m - d = 19$ *(call this equation #2)*
(add equations #1 and #2)
$4m = 32$
$m = 8$

Exercise 6D

1 a $r = \frac{u_2}{u_1} = \frac{1}{2}$
$u_n = u_1 (r)^{n-1}$
$u_7 = 16\left(\frac{1}{2}\right)^6$
$u_7 = 16\left(\frac{1}{64}\right)$
$u_7 = \frac{1}{4}$

b $r = \frac{u_2}{u_1} = -3$
$u_n = u_1 (r)^{n-1}$
$u_7 = -4(-3)^6$
$u_7 = -4(729)$
$u_7 = -2916$

c $r = \frac{u_2}{u_1} = 10$
$u_n = u_1 (r)^{n-1}$
$u_7 = 1(10)^6$
$u_7 = 1000000$

d $r = 0.4$
$u_n = u_1 (r)^{n-1}$
$u_7 = 25(0.4)^6$
$u_7 = 25(0.004096)$
$u_7 = 0.1024$

e $r = 3x$
$u_n = u_1 (r)^{n-1}$
$u_7 = 2(3x)^6$
$u_7 = 2(729x^6)$
$u_7 = 1458x^6$

f $r = \frac{u_2}{u_1} = \frac{a^6b^2}{a^7b} = \frac{b}{a}$
$u_n = u_1(r)^{n-1}$
$u_7 = a^7b\left(\frac{b}{a}\right)^6$
$u_7 = a^7b\left(\frac{b^6}{a^6}\right)$
$u_7 = ab^7$

Exercise 6E

1 $u_5 = u_2 (r)^3$
$3.2 = 50r^3$
$r^3 = \frac{3.2}{50} = \frac{8}{125}$
$r = \frac{2}{5} = 0.4$
$u_1(r) = u_2$
$u_1(0.4) = 50$
$u_1 = 125$

2 $u_6 = u_3 (r)^3$
$144 = -18r^3$
$r^3 = \frac{144}{-18} = -8$
$r = -2$
$u_1 (r)^2 = u_3$
$u_1(-2)^2 = -18$
$u_1 = \frac{-18}{4} = -4.5$

3 a We want n such that
$u_1 (r)^{n-1} > 1000$
$u_1 = 16$ and $r = 1.5$, so
$16(1.5)^{n-1} > 1000$
$(1.5)^{n-1} > 62.5$
$n = 12$

this equation can be solved using the table on the GDC, or by using logarithms

b $u_1 (r)^{n-1} > 1000.$
$u_1 = 1,\ r = 2.4,$
$1(2.4)^{n-1} > 1000$
$(2.4)^{n-1} > 1000$
$n = 9$

c $u_1 (r)^{n-1} > 1000.\ u_1 = 112,\ r = \frac{-168}{112},$
$1(2.4)^{n-1} > 1000$
$(2.4)^{n-1} > 1000$
$n = 9$

d $u_1(r)^{n-1} > 1000$. $u_1 = 50$, $r = \frac{55}{50}$, $= 1.1$
$50(1.1)^{n-1} > 1000$
$(1.1)^{n-1} > 20$
$n = 33$

4 We know $u_3 = u_1 r^2$, so $9r^2 = 144$
$r^2 = 16$
$r = \pm 4$

If $r = 4$	If $r = -4$
$u_2 = u_1(r)$	$u_2 = u_1(r)$
$u_2 = 9(4) = 36$	$u_2 = 9(-4) = -36$

5 $18r^2 = 40.5$
$r^2 = \frac{40.5}{18} = 2.25$
$r = \pm 1.5$

If $r = 1.5$	If $r = -1.5$
$p = 18(1.5) = 27$	$p = 18(-1.5) = -27$

6 $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}$
$\frac{4x+4}{7x-2} = \frac{3x}{4x+4}$
$(4x + 4)(4x + 4) = (3x)(7x - 2)$
$16x^2 + 32x + 16 = 21x^2 - 6x$
$5x^2 - 38x - 16 = 0$
$x = 8$

This quadratic equation can be solved using the polynomial root finder on your GDC. There are 2 roots, but the question asks for the positive value of x.

Exercise 6F

1 a $\sum_{n=1}^{8} n$

b Write the series as $3^2 + 4^2 + 5^2 + 6^2 + 7^2$

Then sum is $\sum_{x=3}^{7} x^n$

c Write the series as
$[29 - 2(1)] + [29 + 2(2)] + [29 - 2(3)]$
$+ [29 - 2(4)] + [29 - 2(5)] + [29 - 2(6)]$

Then sum is $\sum_{x=1}^{6} 29 - 2x$

d Write the series as $240\left(\frac{1}{2}\right)^0 + 240\left(\frac{1}{2}\right)^1 + 240\left(\frac{1}{2}\right)^2$

$+ 240\left(\frac{1}{2}\right)^3 + 240\left(\frac{1}{2}\right)^4 + 240\left(\frac{1}{2}\right)^5$

Then sum is $\sum_{x=1}^{6} 240\left(\frac{1}{2}\right)^{n-1}$

e Write the series as $(4 + 1)\,x + (4 + 2)\,x + (4 + 3)\,x$
$+ (4 + 4)\,x + (4 + 5)\,x + (4 + 6) + x$

The sum is $\sum_{n=1}^{6} (4 + n)x$

f Write the series as $(3(1) + 1) + (3(2) + 1) +$
$(3(3) + 1) + (3(4) + 1) + \cdots + (3(8) + 1)$

The sum is $\sum_{n=1}^{8} 3n + 1$

g Write the series as $3^0 + 3^1 + 3^2 + 3^3 + \cdots + 3$

The sum is $\sum_{n=1} 3^{n-1}$

h The sum is $\sum_{n=1}^{5} na^n$

2 a $\sum_{n=1}^{8}(3n + 1) = (3(1) + 1) + (3(2) + 1) + (3(3) + 1)$
$+ (3(4) + 1) + (3(5) + 1) + (3(6) + 1)$
$+ (3(7) + 1) + (3(8) + 1)$
$= 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25$

b $\sum_{a=1}^{5} 4^a = 4^1 + 4^2 + 4^3 + 4^4 + 4^5$
$= 4 + 16 + 64 + 256 + 1024$

c $\sum_{r=3}^{7}(5(2^r)) = 5(8) + 5(16) + 5(32) + 5(64) + 5(128)$
$= 40 + 80 + 160 + 320 + 640$

d $\sum_{n=5}^{a} x^n = x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11}$

3 a $\sum_{n=1}^{a}(8n - 5) = (8(1) - 5) + (8(2) - 5) + (8(3) - 5)$
$+ (8(4) - 5) + (8(5) - 5) + (8(6) - 5)$
$+ (8(7) - 5) + (8(8) - 5) + (8(9) - 5)$
$= 3 + 11 + 19 + 27 + 35 + 43 + 51$
$+ 59 + 67 = 315$

b $\sum_{r=1}^{5}(3^r) = 3^1 + 3^2 + 3^3 + 3^4 + 3^5$
$= 3 + 9 + 27 + 81 + 243$
$= 363$

c $\sum_{m=1}^{7} m^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$
$= 1 + 4 + 9 + 16 + 25 + 36 + 49$
$= 140$

d $\sum_{x=4}^{10} 7x - 4 = (7(4) - 4) + (7(5) - 4) + (7(6) - 4)$
$+ (7(7) - 4) + (7(8) - 4) + (7(9) - 4)$
$+ (7(10) - 4)$
$= 24 + 31 + 38 + 45 + 52 + 59 + 66$
$= 315$

Exercise 6G

1 $S_{12} = \frac{12}{2}\big(2(3) + (12 - 1)(3)\big)$
$S_{12} = 6(6 + 33) = 234$

2 $S_{18} = \frac{18}{2}\big(2(2.6) + (18 - 1)(0.4)\big)$
$S_{18} = 9(5.2 + 6.8) = 108$

3 $S_{27} = \frac{27}{2}\big(2(100) + (27 - 1)(-6)\big)$
$S_{27} = 13.5(200 - 156) = 594$

4 $S_{16} = \frac{16}{2}\big(2(2 - 5x) + (16 - 1)(1 + x)\big)$
$S_{16} = 8(4 - 10x + 15 + 15x)$
$S_{16} = 8(19 + 5x) = 40x + 152$

5 $u_1 = 120$, $d = -4$. we know $u_1 + (n - 1)d = u_n$, so
$120 + (n - 1)(-4) = 28$
$124 - 4n = 28$
$4n = 96$
$n = 24$
$S_{24} = \frac{24}{2}(120 + 28)$
$S_{24} = 12(148) = 1776$

6 $u_1 = 15$ and $d = 7$. From $u_n = u_1 + (n-1)d$,
$15 + (n-1)(7) = 176$
$7n + 8 = 176$
$7n = 168$
$n = 24$
$S_{24} = \frac{24}{2}(15 + 176)$
$S_{24} = 12(191) = 2292$

Exercise 6H

1 We know $S_n = \frac{n}{2}(2u_1 + (n-1)d)$. Here, $u_1 = 4$ so
$S_{30} = \frac{30}{2}(2(4) + 29d) = 1425$
$15(8 + 29d) = 1425$
$120 + 435d = 1425$
$435d = 1305$
$d = 3$

2 a Using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ with $u_1 = 1$ and $d = 6$,
$S_n = \frac{n}{2}(2(1) + (n-1)(6))$
$S_n = \frac{n}{2}(6n - 4)$
$3n^2 - 2n$

b $3n^2 - 2n = 833$
$3n^2 - 2n = 833 = 0$
(use polynomial equation solver on GDC)
$n = 17$

3 a Using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ with $u_1 = -30$ and $d = 3.5$,
$S_n = \frac{n}{2}(2(-30) + (n-1)(3.5))$
$S_n = \frac{n}{2}(3.5n - 63.5)$
$1.75n^2 - 31.75n$

b $1.75n^2 - 31.75n = 105$
$1.75n^2 - 31.75n - 105 = 0$
(use polynomial equation solver on GDC)
$n = 21$

4 a If we write as an arithmetic progression,
$u_1 = 500$, $u_2 = 600$, $u_3 = 700$, ...
December 2012 will be the 12th month. Using $u_n = u_1 + (n-1)d$ with $u_1 = 500$ and $d = 100$, we see that for $n = 12$,
$u_{12} = 500 + (12 - 1)100$
$= 500 + 1100$
$= 1600$

b Using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$
$S_{12} = \frac{12}{2}(1000 + (12 - 1)100)$
$= 12600$

5 Second term given by $u_2 = u_1 + d$
5th term given by $u_5 = u_1 + 4d$.
we're told $u_2 = 4u_5$, so
$u_1 + d = 4(u_1 + 4d)$
$3u_1 + 15d = 0$. (equation 1)

Also, using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$, since we know $S_{10} = -20$
we have $-20 = \frac{10}{2}(2u_1 + 9d)$
$-20 = 10u_1 + 45d$. (equation 2)
(use simultaneous equation solver on GDC)
$a = -20$, $d = 4$.

6 Since $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ and $S_{12} = 10S_3$,
$\frac{12}{2}(2(5) + 11d) = 10\left(\frac{3}{2}(2(5) + 2d)\right)$
$6(10 + 11d) = 15(10 + 2d)$
$60 + 66d = 150 + 30d$
$36d = 90$
$d = 2.5$
$S_{20} = \frac{20}{2}(2(5) + 19(2.5))$
$S_{20} = 10(10 + 47.5)$
$S_{20} = 575$

Exercise 6I

1 a $u_1 = 0.5$ and $r = 3$. Using $S_n = \frac{u_1(r^n - 1)}{r - 1}$,
$S_{12} = \frac{0.5(3^{12} - 1)}{3 - 1}$
$S_{12} = \frac{265720}{2} = 132\,860$

b $S_{12} = \frac{0.3(2^{12} - 1)}{2 - 1}$
$S_{12} = \frac{0.3(4095)}{1} = 1228.5$

c $S_{12} = \frac{64(1 - (-0.5)^{12})}{1 - (-0.5)} = 42.656\,25$

d $S_{12} = \frac{(x+1)(2^{12} - 1)}{2 - 1}$
$S_{12} = \frac{(x+1)(4095)}{1} = 4095x + 4095$

2 a $S_{20} = \frac{0.25(3^{20} - 1)}{3 - 1}$
$S_{20} = \frac{0.25(3486784400)}{2} = 435\,848\,050$

b $S_{20} = \frac{\frac{16}{9}\left(\left(\frac{3}{2}\right)^{20} - 1\right)}{\frac{3}{2} - 1} \approx 11819.58$

c $S_{20} = \frac{3(1 - (-2)^{20})}{1 - (-2)}$
$S_{20} = \frac{3(-1048575)}{3} = -1048\,575$

d $S_{20} = \frac{(\log a)((2)^{20} - 1)}{2 - 1}$
$S_{20} = \frac{1048575(\log a)}{1} = \log(a^{1\,048\,575})$

3 a i $1024\left(\frac{3}{2}\right)^{n-1} = 26\,244$
$\left(\frac{3}{2}\right)^{n-1} = \frac{6561}{256}$
$n - 1 = 8$
$n = 9$

This type of equation can be solved using logarithms.

ii $S_9 = \dfrac{1024\left(\left(\frac{3}{2}\right)^9 - 1\right)}{\frac{3}{2} - 1} = 76\,684$

b **i** $2.7(4)^{n-1} = 2764.8$

$(4)^{n-1} = 1024$

$n - 1 = 5$

$n = 6$

ii $S_6 = \dfrac{2.7\left((4)^6 - 1\right)}{4 - 1} = \dfrac{2.7(4095)}{3} = 3685.5$

c **i** $\dfrac{125}{128}\left(\dfrac{2}{5}\right)^{n-1} = \dfrac{1}{625}$

$\left(\dfrac{2}{5}\right)^{n-1} = \dfrac{128}{78125}$

$n - 1 = 7$

$n = 8$

ii $S_8 = \dfrac{\frac{125}{128}\left(1 - \left(\frac{2}{5}\right)^8\right)}{1 - \frac{2}{5}} = 1.6265375$

d **i** $590.49\left(\dfrac{1}{3}\right)^{n-1} = 0.01$

$\left(\dfrac{1}{3}\right)^{n-1} = \dfrac{0.01}{590.49}$

$n - 1 = 10$

$n = 11$

ii $S_{11} = \dfrac{590.49\left(1 - \left(\frac{1}{3}\right)^{11}\right)}{1 - \frac{1}{3}} = 885.73$

Exercise 6J

1 **a** $u_1 = 25.6$ $r = 1.5$

$S_n = \dfrac{25.6\left((1.5)^n - 1\right)}{1.5 - 1} = \dfrac{25.6\left((1.5)^n - 1\right)}{0.5}$

$= 51.2\left((1.5)^n - 1\right) > 400$

$S_5 = 337.6,\ S_6 = 532$

$n = 6$

You can get these values from the tables on your GDC.

b $S_n = \dfrac{14\left(1 - (-3)^n\right)}{1 - (-3)} = \dfrac{14\left(1 - (-3)^n\right)}{4} = 3.5\left(1 - (-3)^n\right) > 400$

$S_4 = -280,\ S_5 = 854$

$n = 5$

c $S_n = \dfrac{\frac{2}{3}\left(\left(\frac{4}{3}\right)^n - 1\right)}{\frac{4}{3} - 1} = \dfrac{\frac{2}{3}\left(\left(\frac{4}{3}\right)^n - 1\right)}{\frac{1}{3}} = 2\left(\left(\frac{4}{3}\right)^n - 1\right) > 400$

$S_{18} = 353.75\ldots,\ S_{19} = 471.005\ldots$

$n = 19$

d $S_n = \dfrac{0.02\left((10)^n - 1\right)}{10 - 1} = \dfrac{0.02\left((10)^n - 1\right)}{9} > 400$

$S_5 = 222.22,\ S_6 = 2222.22$

$n = 6$

2 $u_8 = u_3 r^{(8-3)} = u_3 r^5.$

$1.2r^5 = 291.6$

$r^5 = 243$

$r = 3$

In order to find S_{10}, we must first find u_1. Now $u_1 r^2 = u_3$, so $u_1(3^2) = 1.2$

$u_1 = \dfrac{2}{15}$ So substituting $u_1 = \dfrac{2}{15}$ and $r = 3$ into

$S_{10} = \dfrac{u_1(r^{10} - 1)}{r - 1}$, we have

$S_{10} = \dfrac{\frac{2}{15}\left((3)^{10} - 1\right)}{3 - 1} = \dfrac{\frac{2}{15}\left((3)^{10} - 1\right)}{2} = \dfrac{59048}{15}$

3 $S_4 = \dfrac{u_1(r^4 - 1)}{r - 1} = 20 \rightarrow \dfrac{(r^4 - 1)}{20} = \dfrac{r - 1}{u_1}$

$S_7 = \dfrac{u_1(r^7 - 1)}{r - 1} = 546.5 \rightarrow \dfrac{(r^7 - 1)}{546.5} = \dfrac{r - 1}{u_1}$

$\dfrac{r^4 - 1}{20} = \dfrac{r^7 - 1}{546.5} \rightarrow 20(r^7 - 1) = 546.5(r^4 - 1)$

$20r^7 - 546.5r^4 + 526.5 = 0$

$n = 3$

4 **a** $r = \dfrac{\left(\frac{1}{8}\right)}{\left(\frac{1}{12}\right)} = \dfrac{12}{8} = \dfrac{3}{2}$

b $S_n = \dfrac{\frac{1}{12}\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1} = \dfrac{\frac{1}{12}\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{1}{2}} = \dfrac{1}{6}\left(\left(\frac{3}{2}\right)^n - 1\right) > 800$

$S_{20} = 554.04\ldots,\ S_{21} = 831.147\ldots$

$n = 21$

5 $S_3 = \dfrac{u_1(r^3 - 1)}{r - 1} = 304 \rightarrow \dfrac{(r^3 - 1)}{304} = \dfrac{r - 1}{u_1}$

$S_6 = \dfrac{u_1(r^6 - 1)}{r - 1} = 1330 \rightarrow \dfrac{(r^6 - 1)}{1330} = \dfrac{r - 1}{u_1}$

$\dfrac{(r^3 - 1)}{304} = \dfrac{(r^6 - 1)}{1330} \rightarrow 304(r^6 - 1) = 1330(r^3 - 1)$

$304r^6 - 1330r^3 + 1026 = 0$

$r = 1.5$

$\dfrac{u_1\left((1.5)^3 - 1\right)}{1.5 - 1} = 304 \rightarrow u_1(2.375) = 152$

$u_1 = 64$

Now using $S_7 = \dfrac{u_1(r^7 - 1)}{r - 1}$ with $u_1 = 64,\ r = 1.5$

$S_7 = \dfrac{64\left((1.5)^7 - 1\right)}{1.5 - 1} = 128\left((1.5)^7 - 1\right)$

$S_7 = 2059$

6 $S_4 = \dfrac{u_1(r^4 - 1)}{r - 1}$ and $S_2 = \dfrac{u_1(r^2 - 1)}{r - 1}$. since $S_4 = 10S_2$,

$\dfrac{u_1(r^4 - 1)}{r - 1} = 10\left(\dfrac{u_1(r^2 - 1)}{r - 1}\right)$ multiply both side by $\dfrac{r - 1}{u_1}$

$r^4 - 1 = 10r^2 - 10$

$r^4 - 10r^2 + 9 = 0$

$r = 3$

Exercise 6K

1 $|r| < 1$ means that a germetiric series will be convergent.

2 a $S_4 = \frac{144\left(1-\left(\frac{1}{3}\right)^4\right)}{\left(1-\frac{1}{3}\right)} = 213.\overline{3}$

$S_7 = \frac{144\left(1-\left(\frac{1}{3}\right)^7\right)}{\left(1-\frac{1}{3}\right)} \approx 215.9$

$S_\infty = \frac{144}{\left(1-\frac{1}{3}\right)} = 216$

b $S_4 = \frac{500\left(1-(0.8)^4\right)}{(1-0.8)} = 1476$

$S_7 = \frac{500\left(1-0.8^7\right)}{(1-0.8)} = 1975.712$

$S_\infty = \frac{500}{(1-0.8)} = 2500$

c $S_4 = \frac{80\left(1-0.1^4\right)}{(1-0.1)} = 88.88$

$S_7 = \frac{80\left(1-(0.1)^7\right)}{(1-0.1)} = 88.8888$

$S_\infty = \frac{80}{(1-0.1)} = 88.\overline{8}$

d $S_4 = \frac{\frac{9}{2}\left(1-\left(\frac{2}{3}\right)^4\right)}{\left(1-\frac{2}{3}\right)} = 10.8\overline{3}$

$S_7 = \frac{\frac{9}{2}\left(1-\left(\frac{2}{3}\right)^7\right)}{\left(1-\frac{2}{3}\right)} \approx 12.71$

$S_\infty = \frac{\left(\frac{9}{2}\right)}{\left(1-\frac{2}{3}\right)} = 13.5$

3 $S_3 = \frac{u_1(1-r^3)}{1-r} = 13 \rightarrow \frac{1-r^3}{13} = \frac{1-r}{u_1}$ (1)

$S_\infty = \frac{u_1}{1-r} = \frac{27}{2} \rightarrow \frac{2}{27} = \frac{1-r}{u_1}$ (2)

Equating (1) and (2)

$\frac{1-r^3}{13} = \frac{2}{27} \rightarrow 1 - r^3 = \frac{26}{27}$

$r^3 = \frac{1}{27} \rightarrow r = \frac{1}{3}$

put $r = \frac{1}{3}$ in S_∞

$\frac{u_1}{\left(1-\frac{1}{3}\right)} = \frac{27}{2} \rightarrow u_1 = 9$

$S_5 = \frac{9\left(1-\left(\frac{1}{3}\right)^5\right)}{\left(1-\frac{1}{3}\right)} = 13.5\left(1-\left(\frac{1}{3}\right)^5\right)$

$S_5 = 13.\overline{4}$

4 $u_3 r^3 = u_6$, so $24r^3 = 3 \rightarrow r^3 = \frac{1}{8} \rightarrow r = \frac{1}{2}$

since $u_1 r^2 = u_3$, $u_1\left(\frac{1}{2}\right)^2 = 24 \rightarrow u_1 = 96$

$S_\infty = \frac{96}{\left(1-\frac{1}{2}\right)} = 192$

5 $u_2 = u_1(r) = 12 \rightarrow r = \frac{12}{u_1}$ (1)

$S_\infty = \frac{u_1}{1-r} = 64 \rightarrow 1 - r = \frac{u_1}{64} \rightarrow r = 1 - \frac{u_1}{64}$ (2)

$(1) = (2) \Rightarrow 1 - \frac{u_1}{64} = \frac{12}{u_1} \rightarrow u_1 - \frac{(u_1)^2}{64} = 12$

$\rightarrow 64u_1 - (u_1)^2 = 768$

$(u_1)^2 - 64u_1 + 768 = 0$

$u_1 = 16$ *or* 48

6 $r = 0.4$, $S_\infty = \frac{u_1}{1-r} = 250$

$\frac{u_1}{1-0.4} = 250$

$u_1 = 150$

7 $S_5 = \frac{u_1(1-r^5)}{1-r} = 3798$ (1)

$S_\infty = \frac{u_1}{1-r} = 4374 \rightarrow \frac{u_1(1-r^5)}{1-r} = 4374(1-r^5)$ (2)

$(1) = (2) \Rightarrow 4374(1 - r^5) = 3798$

$4374r^5 = 576$

$r^5 = \frac{576}{4374} = \frac{32}{243} \rightarrow r = \frac{2}{3}$

$S_\infty = \frac{u_1}{\left(1-\left(\frac{2}{3}\right)\right)} = 4374 \rightarrow u_1 = 1458$

$S_7 = \frac{1458\left(1-\left(\frac{2}{3}\right)^7\right)}{\left(1-\left(\frac{2}{3}\right)\right)} = 4374\left(1-\left(\frac{2}{3}\right)^7\right)$

$S_7 = 4118$

Exercise 6L

1 $u_6 = u_1 + 5d$, $u_4 = u_1 + 3d$. Since $u_6 = 3u_4$, we have $u_1 + 5d = 3(u_1 + 3d) \rightarrow u_1 + 5d = 3(u_1 + 9d)$

$\rightarrow 2u_1 + 4d = 0$

$u_8 = u_1 + 7d = 50$

$u_1 = -20$

Use simultaneous equation solver on GDC.

2 a $u_1 = 20$

$u_5 = u_1 + 4d = 12 + 4d = 15 \rightarrow 4d = 3 \rightarrow d = 0.75$

$u_{20} = 12 + 19(0.75) = 26.25$

b $12 + (n - 1)(0.75) = 100 \rightarrow 0.75n - 0.75 = 88$

$0.75n = 88.75 \rightarrow n = 118.\overline{3}$

$n = 119$

3 a $2500(1.06)^8$

\$3984.62

b $2500(1.015)^{32}$

\$4025.81

c $2500(1.005)^{96}$

\$4035.36

4 On GDC, let $Y1 = 12x - 7$, and let $Y2 = 0.3(1.2)^{x-1}$

Using table, when x is 41, $Y1 = 485$, and $Y2 \approx 440.93$

When x is 42, $Y1 = 497$, and $Y2 \approx 529.12$

$n = 42$

5 For arithmetic sequence, $S_n = \frac{n}{2}(75 + (n-1)100)$.

For geometric sequence, $S_n = \frac{6(1.5^n - 1)}{1.5 - 1}$.

On GDC, let $Y1 = \frac{6(1.5^x - 1)}{1.5 - 1}$, and

let $Y2 = \frac{x}{2}(2(75) + 100(x-1))$

Using table, when x is 17, $Y1 \approx 11\,811$, and $Y2 = 14\,875$

When x is 18, $Y1 \approx 17\,723$, and $Y2 = 16\,650$

$n = 18$

6 $200(1.05)^3 \approx 232$

7 $275\,000(1.031)^n = 500\,000$

$(1.031)^n = \frac{20}{11}$ — Use logarithms or other GDC methods.

$n \approx 19.6$ years

8 **a** $S_1 = 3(1)^2 - 2(1) = 1$

$S_2 = 3(2)^2 - 2(2) = 8$

$S_3 = 3(3)^2 - 2(3) = 21$

b $u_1 = S_1 = 1$

$u_2 = S_2 - S_1 = 8 - 1 = 7$

$u_3 = S_3 - S_2 = 21 - 8 = 13$

c $d = 6$

$u_n = 1 + (n-1)(6) = 6n - 5$

9 **a** $S_1 = 2^{1+2} - 4 = 4$

$S_2 = 2^{2+2} - 4 = 12$

$S_3 = 2^{3+2} - 4 = 28$

b $u_1 = S_1 = 4$

$u_2 = S_2 - S_1 = 12 - 4 = 8$

$u_3 = S_3 - S_2 = 28 - 12 = 16$

c $r = \frac{u_2}{u_1} = \frac{8}{4} = 2$

$S_n = \frac{4(2^n - 1)}{2 - 1} = 4(2^n - 1)$

10 After n months, species A has $1200(1.0125)^n$ spiders, species B has $50000 - (175)n$ spiders.

On GDC, let $Y1 = 12\,000(1.0125)^x$, and let $Y2 = 50\,000 - 175x$.

Using graph (intersect) or solver $(Y1 - Y2 = 0)$, $x \approx 86.039$.

approx. 86 months

Exercise 6M

1 $\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{120}{6(2)} = 10$

2 $\binom{8}{2} = \frac{8!}{2!(8-2)!} = \frac{40\,320}{2(720)} = 28$

3 $\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{5040}{6(24)} = 35$

4 $\binom{9}{6} = \frac{9!}{6!(9-6)!} = \frac{362\,880}{720(6)} = 84$

5 $\binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{720}{24(2)} = 15$

6 $\binom{10}{3} = \frac{10!}{3!(10-3)!} = \frac{3\,628\,800}{6(5040)} = 120$

Exercise 6N

1 $\binom{5}{0}(y)^5(3)^0 + \binom{5}{1}(y)^4(3)^1 + \binom{5}{2}(y)^3(3)^2 + \binom{5}{3}(y)^2(3)^3 + \binom{5}{4}(y)^1(3)^4 + \binom{5}{5}(y)^0(3)^5$

$(1)(y^5)(1) + (5)(y^4)(3) + (10)(y^3)(9) + (10)(y^2)27 + (5)(y)(81) + (1)(1)(243)$

$y^5 + 15y^4 + 90y^3 + 270y^2 + 405y + 243$

2 $\binom{4}{0}(2b)^4(-1)^0 + \binom{4}{1}(2b)^3(-1)^1 + \binom{4}{2}(2b)^2(-1)^2 + \binom{4}{3}(2b)^1(-1)^3 + \binom{4}{4}(2b)^0(-1)^4$

$(1)(16b^4)(1) + (4)(8b^3)(-1) + (6)(4b^2)(1) + (4)(2b)(-1) + (1)(1)(1)$

$16b^4 - 32b^3 + 24b^2 - 8b + 1$

3 $\binom{6}{0}(3a)^6(2)^0 + \binom{6}{1}(3a)^5(2)^1 + \binom{6}{2}(3a)^4(2)^2 + \binom{6}{3}(3a)^3(2)^3 + \binom{6}{4}(3a)^2(2)^4 + \binom{6}{5}(3a)^1(2)^5 + \binom{6}{6}(3a)^0(2)^6$

$(1)(729a^6)(1) + (6)(243a^5)(2) + (15)(81a^4)(4) + (20)(27a^3)(8) + (15)(9a^2)(16) + (6)(3a)(32) + (1)(1)(64)$

$729a^6 + 2916a^5 + 4860a^4 + 4320a^3 + 2160a^2 + 576a + 64$

4 $\binom{3}{0}(x^2)^3\left(\frac{2}{x}\right)^0 + \binom{3}{1}(x^2)^2\left(\frac{2}{x}\right)^1 + \binom{3}{2}(x^2)^1\left(\frac{2}{x}\right)^2 + \binom{3}{3}(x^2)^0\left(\frac{2}{x}\right)^3$

$(1)(x^6)(1) + (3)(x^4)\left(\frac{2}{x}\right) + (3)(x^2)\left(\frac{4}{x^2}\right) + (1)(1)\left(\frac{8}{x^3}\right)$

$x^6 + 6x^3 + 12 + \frac{8}{x^3}$

5 $\binom{8}{0}x^8y^0 + \binom{8}{1}x^7y^1 + \binom{8}{2}x^6y^2 + \binom{8}{3}x^5y^3 + \binom{8}{4}x^4y^4 + \binom{8}{5}x^3y^5 + \binom{8}{6}x^2y^6 + \binom{8}{7}x^1y^7 + \binom{8}{8}x^0y^8$

$x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

6 $\binom{4}{0}(3a)^4(-2b)^0 + \binom{4}{1}(3a)^3(-2b)^1 + \binom{4}{2}(3a)^2(-2b)^2 + \binom{4}{3}(3a)^1(-2b)^3 + \binom{4}{4}(3a)^0(-2b)^4$

$(1)(81a^4)(1) + (4)(27a^3)(-2b) + (6)(9a^2)(4b^2) + (4)(3a)(-8b^3) + (1)(1)(16b^4)$

$81a^4 - 216a^3b + 216a^2b^2 - 96ab^3 + 16b^4$

7 $\binom{5}{0}(3c)^5\left(\frac{2}{d}\right)^0+\binom{5}{1}(3c)^4\left(\frac{2}{d}\right)^1+\binom{5}{2}(3c)^3\left(\frac{2}{d}\right)^2+\binom{5}{3}(3c)^2\left(\frac{2}{d}\right)^3$

$+\binom{5}{4}(3c)^1\left(\frac{2}{d}\right)^4+\binom{5}{5}(3c)^0\left(\frac{2}{d}\right)^5$

$(1)(243c^5)(1)+(5)(81c^4)\left(\frac{2}{d}\right)+(10)(27c^3)\left(\frac{4}{d^2}\right)$

$+(10)(9c^2)\left(\frac{8}{d^3}\right)+(5)(3c)\left(\frac{16}{d^4}\right)^4+(1)(1)\left(\frac{32}{d^5}\right)$

$243c^5+\frac{810c^4}{d}+\frac{1080c^3}{d^2}+\frac{720c^2}{d^3}$

$+\frac{240c}{d^4}+\frac{32}{d^5}$

8 $\binom{3}{0}(4x^2)^3\left(\frac{1}{2y}\right)^0+\binom{3}{1}(4x^2)^2\left(\frac{1}{2y}\right)^1$

$+\binom{3}{2}(4x^2)^1\left(\frac{1}{2y}\right)^2+\binom{3}{3}(4x^2)^0\left(\frac{1}{2y}\right)^3$

$(1)(64x^6)(1)+(3)(16x^4)\left(\frac{1}{2y}\right)$

$+(3)(4x^2)\left(\frac{1}{4y^2}\right)+(1)(1)\left(\frac{1}{8y^3}\right)$

$64x^6+\frac{24x^4}{y}+\frac{3x^2}{y^2}+\frac{1}{8y^3}$

Exercise 6O

1 $\binom{7}{2}(x)^5(-4)^2=(21)(x^5)(16)=336x^5$

2 $\binom{5}{1}(4y)^4(-1)^1=(5)(256y^4)(-1)=-1280y^4$

3 $\binom{6}{4}(2a)^2(-3b)^4=(15)(4a^2)(81b^4)=4860a^2b^4$

4 $\binom{9}{9}(x)^0(-2)^9=(1)(1)(-512)=-512$

5 $\binom{6}{3}(px)^3(1)^3=(20)(p^3x^3)(1)=20p^3x^3$

$20p^3x^3=160x^3$

$20p^3=160\rightarrow p^3=8$

$p=2$

6 $\binom{7}{2}(3x)^5(q)^2=(21)(243x^5)(q^2)=5103x^5q^2$

$5103q^2x^5=81648x^5$

$5103q^2=81648\rightarrow q^2=16$

$q=\pm4$

7 $\binom{8}{4}(4x)^4\left(\frac{1}{x}\right)^4=(70)(256x^4)\left(\frac{1}{x^4}\right)=17920$

8 $\binom{6}{4}(2x^2)^2\left(-\frac{3}{x}\right)^4=(15)(4x^4)\left(\frac{81}{x^4}\right)=4860$

9 $\binom{n}{3}(x)^3(1)^{n-3}=\binom{n}{3}x^3$

$\binom{n}{2}(x)^2(1)^{n-2}=\binom{n}{2}x^2$

$\binom{n}{3}=\frac{n(n-1)(n-2)(n-3)(n-4)\dots(1)}{3!(n-3)(n-4)\dots(1)}$

$=\frac{n(n-1)(n-2)}{3!}=\frac{n(n-1)(n-2)}{6}$

$\binom{n}{2}=\frac{n(n-1)(n-2)(n-3)\dots(1)}{2!(n-2)(n-3)\dots(1)}$

$=\frac{n(n-1)}{2!}=\frac{n(n-1)}{2}$

$\binom{n}{3}=2\binom{n}{2}\rightarrow\frac{n(n-1)(n-2)}{6}=2\left(\frac{n(n-1)}{2}\right)$

$\frac{n(n-1)(n-2)}{6}=n(n-1)\rightarrow n(n-1)(n-2)$

$=6n(n-1)$

$n-2=6$

$n=8$

10 $\binom{n}{3}(x)^3(2)^{n-3}=\binom{n}{3}\left(\frac{2^n}{8}\right)x^3$

$\binom{n}{4}(x)^4(2)^{n-4}=\binom{n}{4}\left(\frac{2^n}{16}\right)x^4$

$\binom{n}{3}=\frac{n(n-1)(n-2)(n-3)(n-4)\dots(1)}{3!(n-3)(n-4)\dots(1)}$

$=\frac{n(n-1)(n-2)}{3!}=\frac{n(n-1)(n-2)}{6}$

$\binom{n}{4}=\frac{n(n-1)(n-2)(n-3)(n-4)\dots(1)}{4!(n-4)(n-5)\dots(1)}$

$=\frac{n(n-1)(n-2)(n-3)}{4!}=\frac{n(n-1)(n-2)(n-3)}{24}$

$\binom{n}{3}\left(\frac{2^n}{8}\right)=2\binom{n}{4}\left(\frac{2^n}{16}\right)\rightarrow\left(\frac{n(n-1)(n-2)}{6}\right)\left(\frac{2^n}{8}\right)$

$=2\left(\frac{n(n-1)(n-2)(n-3)}{24}\right)\left(\frac{2^n}{16}\right)$

$\left(\frac{n(n-1)(n-2)}{48}\right)(2^n)=\left(\frac{n(n-1)(n-2)(n-3)}{192}\right)(2^n)$

$\left(\frac{n(n-1)(n-2)}{48}\right)=\left(\frac{n(n-1)(n-2)(n-3)}{192}\right)$

$\rightarrow 4(n(n-1)(n-2))=(n(n-1)(n-2)(n-3))$

$n-3=4$

$n=7$

Review exercise

1 **a** $d=u_2-u_1=7-3=4$

b $u_n=3+70(4)=283$

c $3+4(n-1)=99\rightarrow 4n-1=99$

$4n=100$

$n=25$

2 **a** $r=\frac{u_2}{u_1}=\frac{16}{64}=\frac{1}{4}$

b $u_4=64\left(\frac{1}{4}\right)^3=1$

c $S_\infty=\frac{64}{\left(1-\frac{1}{4}\right)}=\frac{64}{\left(\frac{3}{4}\right)}=\frac{256}{3}$

3 **a** $u_6=u_1+5d=25$

$u_{12}=u_1+11d=49$

$6d=24$

$d=4$

Subtract the first equation from the second.

b $u_1 + 5(4) = 25$
$u_1 + 5$

4 **a** $u_3 = 22 + 2d = 38 \to 2d = 16 \to d = 8$
$u_2 = x = 22 + 8 = 30$

b $u_{31} = 22 + 30(8) = 262$

5 $\sum_{a=1}^{4}(3^a) = 3^1 + 3^2 + 3^3 + 3^4 = 3 + 9 + 27 + 81 = 120$

6 **a** $\frac{1}{4}$

b $S_\infty = \frac{800}{\left(1-\frac{1}{4}\right)} = \frac{800}{\left(\frac{3}{4}\right)} = \frac{3200}{3}$

7 Since $r = \frac{u_2}{u_1} = \frac{u_3}{u_2}, \frac{12}{x} = \frac{9x}{12} \to 9x^2 = 144$
$x^2 = 16$
$x = \pm 4$

8 $\binom{5}{2}(2x)^3(3)^2 = 10(8x^3)(9) = 720x^3$

9 **a** $u_1 = 3, d = 2, 3 + 2(n-1) = 35 \to$
$2n + 1 = 35 \to 2n = 34$
$n = 17$

b $S_{17} = \frac{17}{2}(2(3) + 16(2)) = 8.5(6 + 32) = 323$

Review exercise

1 **a** $S_{25} = \frac{25}{2}(2(4) + 24(d)) = 1000$
$12.5(8 + 24d) = 100 + 300d = 1000$
$300d = 900$
$d = 3$

b $u_{17} = 4 + 16(3) = 52$

2 **a** $u_{63} = 3 + 62(1.5) = 96$

b $\frac{n}{2}(2(3) + (n-1)(1.5)) = 840$
$\frac{n}{2}(1.5n + 4.5) = 840$
$1.5n^2 + 4.5n = 1680$
$1.5n^2 + 4.5n - 1680 = 0$
$n = 32$

Use quadratic equation solver on GDC.

3 **a** $u_{10} = u_1 + 9d = 25$
$S_{10} = \frac{10}{2}(2u_1 + 9d) = 160$
$= \frac{10}{2}(u_1 + [u_1 + 9d]) = 160$
using $u_1 + 9d = 25$, we see
$\frac{10}{2}(u_1 + 25) = 160 \to 5u_1 + 125 = 160$
$5u_1 = 35$
$u_1 = 7$
$7 + 9d = 25$
$9d = 18$
$d = 2$

b $S_{24} = \frac{24}{2}(2(7) + 23(2)) = 720$

4 **a** $u_1 = 3, u_6 = u_1r^5 = 96.$
$3r^5 = 96$
$r^5 = 32$
$r = 2$

b $u_n = 3(2^{n-1})$, so $3(2^{n-1}) > 3000$
$2^{n-1} > 1000$
$n - 1 > 9.97$
$n > 10.97$
$n = 11$

5 arithmetic sequence: $u_n = 28 + 50(n-1)$
geometric sequence: $u_n = 1(1.5)^{n-1}$
On GDC, let $y_1 = 28 + 50(x-1)$ and $y_2 = (1.5)^{x-1}$.
Using table, we see y_2 becomes bigger than y_1 when $n = 18$.

6 $u_3 = u_1(r^2) = 45 \to u_1 = \frac{45}{r^2}$
$S_7 = \frac{u_1(1-r^7)}{1-r} = 2735 \to u_1 = \frac{2735(1-r)}{(1-r^7)}$
$\frac{45}{r^2} = \frac{2735(1-r)}{(1-r^7)} \to 45(1-r^7) = 2735r^2(1-r)$
$45 - 45r^7 = 2735r^2 - 2735r^3$
$45r^7 - 2735r^3 + 2735r^2 - 45 = 0$
$r = -3$
$u_1 = \frac{45}{(-3)^2} = 5$

7 $\binom{7}{3}\left(\frac{x}{2}\right)^4(-3)^3 = 35\left(\frac{x^4}{16}\right)(-27) = \frac{-945x^4}{16}$

8 $\binom{8}{3}(ax)^5(2)^3 = 56(a^5x^5)(8) = 448a^5x^5 = \frac{7}{16}x^5$
$448a^5 = \frac{7}{16}$
$a^5 = \frac{7}{7168} = \frac{1}{1024}$
$a = \frac{1}{4}$

9 **a** In 2040, $n = 30$.
$3.4(1.016)^{30} \approx 5.4738$
approx. 5.47 million

b $3.4(1.016)^n = 7$
$(1.016)^n = \frac{7}{3.4}$
$n \approx 45.49$
the year will be 2055