# 3 Probability

## Answers

## Skills check

**1** **a** $1-\frac{3}{7}=\frac{7}{7}-\frac{3}{7}=\frac{4}{7}$

**b** $\frac{2}{5}+\frac{5}{7}=\frac{14+25}{35}=\frac{39}{35}=1\frac{4}{35}$

**c** $\frac{1}{5}\times\frac{2}{3}=\frac{2}{15}$

**d** $1-\left(\frac{1}{3}\times\frac{5}{9}\right)=1-\frac{5}{27}=\frac{27-5}{27}=\frac{22}{27}$

**e** $\left(\frac{2}{3}\times\frac{7}{9}\right)+\left(\frac{1}{3}\times\frac{5}{9}\right)=\frac{14}{27}+\frac{5}{27}=\frac{19}{27}$

**f** $\dfrac{\frac{3}{20}}{\frac{7}{20}}=\frac{3}{20}\times\frac{20}{7}=\frac{3}{7}$

**2** **a** $1-0.375=0.625$

**b** $0.65+0.05=0.7$

**c** $0.25\times0.64=0.16$

**d** 50% of 30 = $0.5\times30=15$

**e** 22% of 0.22 = $0.22\times0.22=0.0484$

**f** 12% of 10% of 0.8 = $0.12\times0.1\times0.8=0.0096$

## Exercise 3A

**1** **a** $P(2, 4, 6, 8)=\frac{4}{8}=\frac{1}{2}$

**b** $P(3, 6)=\frac{2}{8}=\frac{1}{4}$

**c** $P(4, 8)=\frac{2}{8}=\frac{1}{4}$

**d** $P(1, 2, 3, 5, 6, 7)=\frac{6}{8}=\frac{3}{4}$ or

$1-P(4, 8)=1-\frac{1}{4}=\frac{3}{4}$

**e** $P(1, 2, 3)=\frac{3}{8}$

**2** $\text{P(defective car)}=\frac{\text{number defective}}{\text{number of cars}}=\frac{30}{150}=\frac{1}{5}$

**3** **a** **i** 0.21

**ii** $0.19+0.14=0.33$

**b** Proportion of 15 year old students = 0.21

Therefore $0.21\times1200=252$ students who are 15.

**4** **a** $\frac{27}{100}=0.27$

**b** No – the frequencies for different numbers are very different

**c** $\frac{15}{100}\times300=45$

**5** **a** $\frac{\text{number of c's}}{\text{number of letters}}=\frac{2}{11}$

**b** $\frac{\text{number of p's}}{\text{number of letters}}=\frac{0}{11}=0$

**c** $\frac{\text{number of vowels}}{\text{number of letters}}=\frac{5}{11}$

**6** P(red) + P(yellow) + P(green) + P(blue) = 1

Let P(yellow) = $x$ so P(green) = $2x$

$0.4+x+2x+0.3=1$

$3x=0.3$

$x=0.1$

Therefore P(green) = 0.2

**7** **a** $\frac{\text{number of even numbers}}{\text{number of possible outcomes}}=\frac{20}{40}=\frac{1}{2}$

**b** {1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31}

**c** $\frac{\text{number that contain digit 1}}{\text{number of possible outcomes}}=\frac{13}{40}$

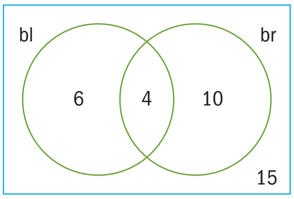
## Exercise 3B

**1** $n$(blond and brown) = 4

$n$(blond and not brown) = 10 – 4 = 6

$n$(brown and not blond) = 14 – 4 = 10

$n$(neither blond or brown) = 35 – (6 + 4 + 10) = 15




P(blond hair or blue eyes) = $\frac{6+4+10}{35}=\frac{20}{35}=\frac{4}{7}$

**2** $n$(French and Malay) = $x$

$n$(F and not M) = $15-x$

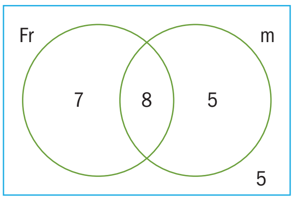
$n$(M and not F) = $13-x$

$n$(neither F or M) = 5

Therefore $x+(15-x)+(13-x)+5=25$

$33-x=25$

$x=8$




P(F and M) = $\frac{8}{25}$

**3** $n$(Aerobics and Gymnastics) = $x$

$n$(A and not G) = $13-x$

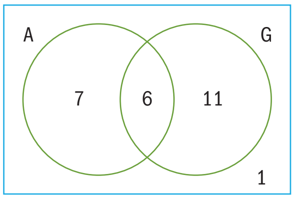
$n$(G and not A) = $17-x$

$n$(neither A or G) = 1

Therefore $x+(13-x)+(17-x)+1=25$

$31 - x = 25$

$x = 6$




**a** P(A and G) = $\frac{6}{25}$

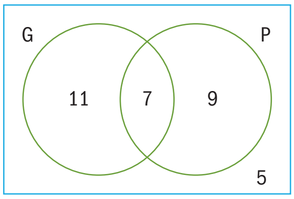
**b** P(G and not A) = $\frac{11}{25}$

**4** $n$(Golf and Piano) = 7

$n$(G and not P) = 18 – 7 = 11

$n$(P and not G) = 16 – 7 = 9

$n$(neither G or P) = 32 – (7 + 11 + 9) = 5




**a** P(G and not P) = $\frac{11}{32}$

**b** P(P and not G) = $\frac{9}{32}$

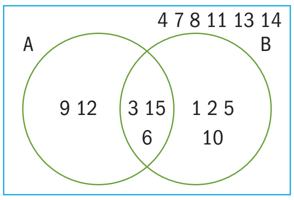
**5** **a** $A$ = {integers that are multiples of 3}

= { 3, 6, 9, 12, 15}

$B$ = {integers that are factors of 30}

= {1, 2, 3, 5, 6, 10, 15}

**b**




**c** **i** P(both a multiple of 3 and a factor of 30) = $\frac{3}{15} = \frac{1}{5}$

**ii** P(Neither a multiple of 3 or a factor of 30) = $\frac{6}{15} = \frac{2}{5}$

**6** $n$(A & B, not C) = 5% − 2% = 3%

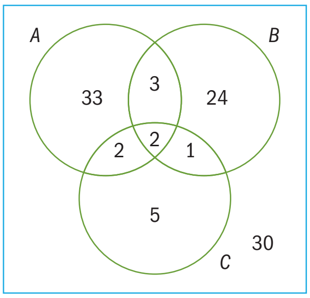
$n$(A & C, not B) = 4% − 2% = 2%

$n$(B & C, not A) = 3% − 2% = 1%

$n$(A, not B or C) = 40% − (2% + 3% + 2%) = 33%

$n$(B, not A or C) = 30% − (2% + 3% + 1%) = 24%

$n$(C, not A or B) = 10% − (2% + 2% + 1%) = 5%




**a** P(only A) = 0.33

**b** P(only B) = 0.24

**c** P(none) = 0.3

## Exercise 3C

**1** **a** $\frac{\text{number that are divisible by 5}}{\text{number of possible outcomes}} = \frac{\text{frequencies of }\{5, 10\}}{\text{number of possible outcomes}}$

$= \frac{34+68}{500} = \frac{102}{500} = \frac{51}{250}$

**b** $\frac{\text{number that are even}}{\text{number of possible outcomes}}$

$= \frac{\text{frequencies of }\{2, 4, 6, 8, 10, 12\}}{\text{number of possible outcomes}}$

$= \frac{6+21+65+63+68+42}{500} = \frac{265}{500} = \frac{53}{100}$

**c** $\frac{\text{number that are divisible by 5 or even}}{\text{number of possible outcomes}}$

$= \frac{\text{frequencies of }\{2,4,5,6,8,10,12\}}{\text{number of possible outcomes}}$

$= \frac{6+21+65+63+68+42+34}{500} = \frac{299}{500}$

or P(sum divisible by 5 ∪ sum even)

= P(sum divisible by 5) + P(sum even)

− P(sum divisible by 5 ∩ sum even)

$= \frac{102}{500} + \frac{265}{500} - \frac{68}{500} = \frac{299}{500}$

**2** **a** P(prime) = $\frac{4}{10} = \frac{2}{5}$ [primes are 2, 3, 5, 7]

**b** P(prime or multiple of 3) = $\frac{4}{10} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$

**c** P(multiple of 3 or 4) = $\frac{3}{10} + \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$

**3** P(camera owner or female)

= P(camera owner) + P(female)

− P(female camera owner)

$= \frac{40}{80} + \frac{50}{80} - \frac{22}{80} = \frac{68}{80} = \frac{17}{20}$

**4** **a** 8 different letters in MATHEMATICS{M, A, T, H, E, I, C, S}. $\frac{8}{26} = \frac{4}{13}$

**b** 9 different letters in TRIGONOMETRY {T, R, I, G, O, N, M, E, Y} $\frac{9}{26}$

**c** {M, T, E, I} $\frac{4}{26} = \frac{2}{13}$

**d** {M, A, T, H, E, I, C, S, R, G, O, N, Y} $\frac{13}{26} = \frac{1}{2}$

**5** **a** P(work of fiction, non-fiction, or both) = 0.40 + 0.30 − 0.20 = 0.5

**b** P(no book) = 1 − 0.5 = 0.5

**6** Let P(local and national) = $x$

P(national and not local) = $\frac{1}{4} - x$

P(local and not national) = $\frac{3}{5} - x$

$\frac{2}{3} = \left(\frac{1}{4} - x\right) + \left(\frac{3}{5} - x\right) + x$

$x = \frac{11}{60}$

**7** **a** $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{1}{4} + \frac{3}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

**b** $P(X \cup Y)' = 1 - P(X \cup Y) = 1 - \frac{1}{2} = \frac{1}{2}$

**8** **a** $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$= 0.2 + 0.5 - 0.1 = 0.6$

**b** $P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.6 = 0.4$

**c** $P(A' \cup B) = 1 - P(A \cap B')$

$= 1 - [P(A) - P(A \cap B)]$

$= 1 - [0.2 - 0.1] = 0.9$

## Exercise 3D

**1** **a** A and B = N **b** A and C = Y

**c** A and D = N **d** A and E = Y

**e** B and E = N **f** C and D = N

**g** B and C = N

**2** Let $P(N \cap M) = x$. If $P(N \cap M) = 0$ then $N$ & $M$ are mutually exclusive.

Now $P(N \cup M) = P(N) + P(M) - P(N \cup M)$, so

$\frac{3}{10} = \frac{1}{5} + \frac{1}{10} - x$

$x = 0$

Therefore yes

**3** $\frac{30}{89} + \frac{27}{89} = \frac{57}{89}$

**4** **a** $\frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$

**b** $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20+15+12}{60} = \frac{47}{60}$

**c** $1 - \frac{47}{60} = \frac{13}{60}$

## Exercise 3E

**1** {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

**a** {HHH, HHT, HTH, THH} $\frac{1}{2}$

**b** {HHH, HHT, THH} $\frac{3}{8}$

**c** {HTH,THT} $\frac{1}{4}$

**2**

| | | BLUE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| RED | 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) |
| | 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) |
| | 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) |
| | 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) |

**a** $\frac{6}{16} = \frac{3}{8}$

| | | BLUE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| RED | 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) |
| | 2 | **(2, 1)** | (2, 2) | (2, 3) | (2, 4) |
| | 3 | **(3, 1)** | **(3, 2)** | (3, 3) | (3, 4) |
| | 4 | **(4, 1)** | **(4, 2)** | **(4, 3)** | (4, 4) |

**b** $\frac{6}{16} = \frac{3}{8}$

| | | BLUE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| RED | 1 | (1, 1) | **(1, 2)** | (1, 3) | (1, 4) |
| | 2 | **(2, 1)** | (2, 2) | **(2, 3)** | (2, 4) |
| | 3 | (3, 1) | **(3, 2)** | (3, 3) | **(3, 4)** |
| | 4 | (4, 1) | (4, 2) | **(4, 3)** | (4, 4) |

**c** $\frac{4}{16} = \frac{1}{4}$

| | | BLUE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| RED | 1 | (1, 1) | **(1, 2)** | (1, 3) | **(1, 4)** |
| | 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) |
| | 3 | (3, 1) | **(3, 2)** | (3, 3) | **(3, 4)** |
| | 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) |

**d** $\frac{9}{16}$

| | | BLUE | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| RED | 1 | **(1, 1)** | **(1, 2)** | (1, 3) | **(1, 4)** |
| | 2 | **(2, 1)** | (2, 2) | **(2, 3)** | (2, 4) |
| | 3 | (3, 1) | **(3, 2)** | (3, 3) | **(3, 4)** |
| | 4 | **(4, 1)** | (4, 2) | **(4, 3)** | (4, 4) |

**3**

| | | Box 1 | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| Box 2 | 2 | (2, 1) | (2, 2) | (2, 3) |
| | 3 | (3, 1) | (3, 2) | (3, 3) |
| | 4 | (4, 1) | (4, 2) | (4, 3) |
| | 5 | (5, 1) | (5, 2) | (5, 3) |

**a** $\frac{2}{12} = \frac{1}{6}$

| | | Box 1 | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| Box 2 | 2 | (2, 1) | **(2, 2)** | (2, 3) |
| | 3 | (3, 1) | (3, 2) | **(3, 3)** |
| | 4 | (4, 1) | (4, 2) | (4, 3) |
| | 5 | (5, 1) | (5, 2) | (5, 3) |

**b** $\frac{3}{12} = \frac{1}{4}$

| | | Box 1 | | |
| --- | --- | --- | --- | --- |
| | | 1 | 2 | 3 |
| Box 2 | 2 | (2, 1) | (2, 2) | **(2, 3)** |
| | 3 | **(3, 1)** | **(3, 2)** | (3, 3) |
| | 4 | (4, 1) | (4, 2) | (4, 3) |
| | 5 | (5, 1) | (5, 2) | (5, 3) |

**c** $\frac{9}{12} = \frac{3}{4}$

| | | Box 1 | | |
| --- | --- | --- | --- | --- |
| | | 1 | 2 | 3 |
| Box 2 | 2 | **(2, 1)** | **(2, 2)** | **(2, 3)** |
| | 3 | **(3, 1)** | **(3, 2)** | **(3, 3)** |
| | 4 | **(4, 1)** | **(4, 2)** | (4, 3) |
| | 5 | **(5, 1)** | (5, 2) | (5, 3) |

**d** $\frac{5}{12}$

| | | Box 1 | | |
| --- | --- | --- | --- | --- |
| | | 1 | 2 | 3 |
| Box 2 | 2 | (2, 1) | (2, 2) | (2, 3) |
| | 3 | (3, 1) | (3, 2) | **(3, 3)** |
| | 4 | (4, 1) | **(4, 2)** | **(4, 3)** |
| | 5 | (5, 1) | **(5, 2)** | **(5, 3)** |

**e** $\frac{8}{12} = \frac{2}{3}$

| | | Box 1 | | |
| --- | --- | --- | --- | --- |
| | | 1 | 2 | 3 |
| Box 2 | 2 | **(2, 1)** | **(2, 2)** | **(2, 3)** |
| | 3 | (3, 1) | **(3, 2)** | (3, 3) |
| | 4 | **(4, 1)** | **(4, 2)** | **(4, 3)** |
| | 5 | (5, 1) | **(5, 2)** | (5, 3) |

**4**

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | (0, 0) | (0, 1) | (0, 2) | (0, 3) | (0, 4) | (0, 5) |
| | 1 | (1, 0) | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) |
| | 2 | (2, 0) | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) |
| | 3 | (3, 0) | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) |
| | 4 | (4, 0) | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) |
| | 5 | (5, 0) | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) |

**a** $\frac{6}{36} = \frac{1}{6}$

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | **(0, 0)** | (0, 1) | (0, 2) | (0, 3) | (0, 4) | (0, 5) |
| | 1 | (1, 0) | **(1, 1)** | (1, 2) | (1, 3) | (1, 4) | (1, 5) |
| | 2 | (2, 0) | (2, 1) | **(2, 2)** | (2, 3) | (2, 4) | (2, 5) |
| | 3 | (3, 0) | (3, 1) | (3, 2) | **(3, 3)** | (3, 4) | (3, 5) |
| | 4 | (4, 0) | (4, 1) | (4, 2) | (4, 3) | **(4, 4)** | (4, 5) |
| | 5 | (5, 0) | (5, 1) | (5, 2) | (5, 3) | (5, 4) | **(5, 5)** |

**b** $\frac{20}{36} = \frac{5}{9}$

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | (0, 0) | (0, 1) | **(0, 2)** | **(0, 3)** | (0, 4) | **(0, 5)** |
| | 1 | (1, 0) | (1, 1) | **(1, 2)** | **(1, 3)** | (1, 4) | **(1, 5)** |
| | 2 | **(2, 0)** | **(2, 1)** | (2, 2) | **(2, 3)** | (2, 4) | **(2, 5)** |
| | 3 | **(3, 0)** | **(3, 1)** | **(3, 2)** | (3, 3) | (3, 4) | **(3, 5)** |
| | 4 | (4, 0) | (4, 1) | (4, 2) | (4, 3) | (4, 4) | **(4, 5)** |
| | 5 | **(5, 0)** | **(5, 1)** | **(5, 2)** | **(5, 3)** | **(5, 4)** | (5, 5) |

**c** $\frac{26}{36} = \frac{13}{18}$

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | **(0, 0)** | **(0, 1)** | **(0, 2)** | **(0, 3)** | **(0, 4)** | **(0, 5)** |
| | 1 | **(1, 0)** | **(1, 1)** | **(1, 2)** | **(1, 3)** | **(1, 4)** | **(1, 5)** |
| | 2 | **(2, 0)** | **(2, 1)** | **(2, 2)** | **(2, 3)** | **(2, 4)** | (2, 5) |
| | 3 | **(3, 0)** | **(3, 1)** | **(3, 2)** | **(3, 3)** | (3, 4) | (3, 5) |
| | 4 | **(4, 0)** | **(4, 1)** | **(4, 2)** | (4, 3) | (4, 4) | (4, 5) |
| | 5 | **(5, 0)** | **(5, 1)** | (5, 2) | (5, 3) | (5, 4) | (5, 5) |

**d** $\frac{13}{36}$

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | (0, 0) | (0, 1) | (0, 2) | (0, 3) | (0, 4) | (0, 5) |
| | 1 | (1, 0) | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) |
| | 2 | (2, 0) | (2, 1) | (2, 2) | (2, 3) | **(2, 4)** | **(2, 5)** |
| | 3 | (3, 0) | (3, 1) | (3, 2) | **(2, 3)** | **(3, 4)** | **(3, 5)** |
| | 4 | (4, 0) | (4, 1) | **(4, 2)** | **(4, 3)** | **(4, 4)** | **(4, 5)** |
| | 5 | (5, 0) | (5, 1) | **(5, 2)** | **(5, 3)** | **(5, 4)** | **(5, 5)** |

**e** $\frac{20}{36} = \frac{5}{9}$

| | | First draw | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | 0 | 1 | 2 | 3 | 4 | 5 |
| Second draw | 0 | (0, 0) | (0, 1) | **(0, 2)** | (0, 3) | **(0, 4)** | (0, 5) |
| | 1 | (1, 0) | (1, 1) | **(1, 2)** | (1, 3) | **(1, 4)** | (1, 5) |
| | 2 | **(2, 0)** | **(2, 1)** | **(2, 2)** | **(2, 3)** | **(2, 4)** | **(2, 5)** |
| | 3 | (3, 0) | (3, 1) | **(3, 2)** | (3, 3) | **(3, 4)** | (3, 5) |
| | 4 | **(4, 0)** | **(4, 1)** | **(4, 2)** | **(4, 3)** | **(4, 4)** | **(4, 5)** |
| | 5 | (5, 0) | (5, 1) | **(5, 2)** | (5, 3) | **(5, 4)** | (5, 5) |

**5** When rolling the dice twice, there are 36 possible outcomes

**a** {(1, 3), (2,4), (3, 1), (4, 2), (5, 5), (5, 6), (6, 5), (6, 6)}; $\frac{8}{36} = \frac{2}{9}$

**b** {(1, 1), (2, 2), (3, 3), (4, 4)}; $\frac{4}{36} = \frac{1}{9}$

**c** {(1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 4), (4, 1), (4, 3)}; $\frac{8}{36} = \frac{2}{9}$

## Exercise 3F

**1** $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$

**2** $P(K) \times P(10) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

**3** $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$

**4** $P(C) \times P(H) = 0.75 \times 0.85 = 0.6375 = 0.638$

**5** **a** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Let $P(B) = x$.
$0.4 = 0.2 + x - 0$
$x = 0.2$
$P(B) = 0.2$

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$
Let $P(B \cap C) = y$.
$0.34 = 0.2 + 0.3 - y$
$y = 0.16$
$P(B \cap C) = 0.16$

**b** $P(B) \times P(C) = 0.2 \times 0.3 = 0.06$
$P(B \cap C) = 0.16$
$P(B \cap C) \neq P(B) \times P(C)$
Not independent

**6** $P(H) \times P(\bar{6}) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

**7** $\left(\frac{1}{9}\right)^5 = \frac{1}{59049}$

**8** $P(H) = \frac{1}{4}$, therefore for 4 hearts $\left(\frac{1}{4}\right)^4 = \frac{1}{256}$

**9** **a** $P(E) = 1 - P(E') = 1 - 0.6 = 0.4$

**b** $P(E) \times P(F) = 0.4 \times 0.6 = 0.24 = P(E \cap F)$

**c** $P(E \cap F) \neq 0$

**d** $P(E \cup F') = P(E) + P(F') - P(E \cap F')$
We know that since $E$ & $F$ are independent,
$P(E \cap F') = P(E) \times P(F') = 0.4 \times 0.4$
$P(E \cup F') = 0.4 + 0.4 - (0.4 \times 0.4) = 0.64$

**10** $P(R_1 \text{ and } B_2 \text{ and } R_3) = \frac{4}{12} \times \frac{8}{12} \times \frac{4}{12} = \frac{2}{27}$

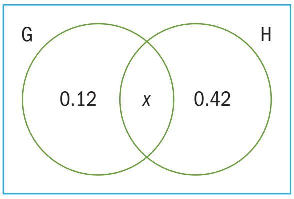
**11** $\{2, 2, 2\}$; $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

**12** **a** $P(A \cap B) = P(A) \times P(B) = 0.9 \times 0.3 = 0.27$
since $A$ & $B$ are independent

**b** $P(A \cap B') = 0.9 \times 0.7 = 0.63$
(since $P(B') = 1 - P(B) = 0.7$)

**c** $P(A \cup B)' = 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - (0.9 + 0.3 - 0.27)$
$= 0.07$

**13**




$G$ & $H$ are independent, so $P(G \cap H) = P(G) \times P(H)$
Now $P(G) = 0.12 + x$, and $P(H) = 0.42 + x$, so
$(0.12 + x)(0.42 + x) = x$
$x^2 - 0.46x + 0.0504 = 0$
$x = 0.18, 0.28$

**14** **a** $P(\text{4 sixes}) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$

**b** $P(\text{4 same}) = 6\left(\frac{1}{6}\right)^4 = \frac{6}{1296} = \frac{1}{216}$

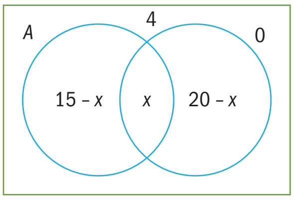
**15** rolling a 'six' on four throws of one dice:
P(rolling a 'six' on four throws of one dice) $= 1 - \left(\frac{5}{6}\right)^4 = 1 - \frac{625}{1296} = \frac{671}{1296} = 0.518$
P(rolling a 'double six' on 24 throws with two dice) $= 1 - \left(\frac{35}{36}\right)^{24} = 1 - 0.5085\ldots = 0.491$

**16** **a** P(not a 5) = 0.9. we require $(0.9)^3 = 0.729$

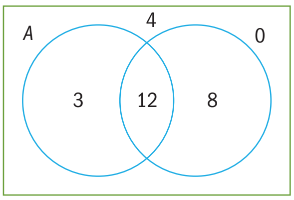
**b** $1 -$ P(none is a 5) $= 1 - 0.729 = 0.271$

## Exercise 3G

**1** Let $n(A \cap D) = x$




$15 - x + x + 20 - x + 4 = 27$
$39 - x = 27$
$x = 12$




**a** P(Drama not Art) $= \frac{8}{27}$

**b** P(Takes at least one of the two subjects)
$= 1 -$ P(takes none) $= 1 - \frac{4}{27} = \frac{23}{27}$

**c** P(Takes both subjects, given that he takes Art)
$= \frac{\frac{12}{27}}{\frac{15}{27}} = \frac{12}{15} = \frac{4}{5}$

**2** **a** $P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.35 = 0.65$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = 0.25 + 0.6 - P(A \cap B)$

$P(A \cap B) = 0.85 - 0.65 = 0.2$

**b** $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.2}{0.6}$

$= \frac{1}{3}$

**c** $P(B'|A') = \frac{P(B' \cap A')}{P(A')}$

$= \frac{0.35}{0.75}$

$= \frac{7}{15}$

**3** $P(R|S) = \frac{P(R \cap S)}{P(S)}$

$= \frac{0.39}{0.48}$

$= 0.8125 = \frac{13}{16}$

**4** **a** $P(E|M') = \frac{P(E \cap M')}{P(M')}$

$= \frac{0.25}{0.75}$

$= \frac{1}{3}$

**b** $P(<15|>5) = \frac{P(<15 \cap >5)}{P(>5)}$

$= \frac{\frac{2}{8}}{\frac{5}{8}}$

$= \frac{2}{5}$

**c** $P(<5|<15) = \frac{P(<5 \cap <15)}{P(<15)}$

$= \frac{\frac{3}{8}}{\frac{5}{8}}$

$= \frac{3}{5}$

**d** $P(\text{between } 10 \text{ and } 20 | \text{between } 5 \text{ and } 25)$

$= \frac{P(\text{between 10 and 20 and between 5 and 25})}{P(\text{between 5 and 25})}$

$= \frac{\frac{2}{8}}{\frac{4}{8}}$

$= \frac{1}{2}$

**5** $P(L|D) = \frac{P(D \cap L)}{P(D)} = \frac{0.61}{0.95} = \frac{61}{95}$

**6** $P(S|T) = \frac{P(T \cap S)}{P(T)} = \frac{0.1}{0.6} = \frac{1}{6}$

**7** **a** P(U and V) = 0 by definition

**b** P(U | V) = 0 by definition

**c** P(U or V) = P(U) + P(V) = 0.26 + 0.37 = 0.63

**8** $\frac{P(\text{Pass both})}{P(\text{Pass first})} = \frac{0.35}{0.52} = 0.673$. Therefore 67.3%

**9** $P(B_1 \text{ and } W_2) = 0.34$; $P(B_1) = 0.47$.

$P(W_2|B_1) = \frac{P(B_1 \text{ and } W_2)}{P(B_1)} = \frac{0.34}{0.47} = \frac{34}{47}$

**10** **a** P(male and left handed) = $\frac{5}{50} = \frac{1}{10}$

| | Left handed | Right handed | Total |
|---|---|---|---|
| Male | 5 | 32 | 37 |
| Female | 2 | 11 | 13 |
| Total | 7 | 43 | 50 |

**b** P(right handed) = $\frac{43}{50}$

| | Left handed | Right handed | Total |
|---|---|---|---|
| Male | 5 | 32 | 37 |
| Female | 2 | 11 | 13 |
| Total | 7 | 43 | 50 |

**c** P(right handed, given that the player selected is female) = $\frac{11}{13}$

| | Left handed | Right handed | Total |
|---|---|---|---|
| Male | 5 | 32 | 37 |
| Female | 2 | 11 | 13 |
| Total | 7 | 43 | 50 |

**11** $P(J|K) = \frac{P(J \cap K)}{P(K)}$

$J$ & $K$ are independent, so $P(J \cap K) = P(J) \times P(K)$

$\therefore P(J|K) = = \frac{P(J) \times \cancel{P(K)}}{\cancel{P(K)}} = P(J)$

so $P(J) = P(J|K) = 0.3$

**12** Let T be the event the neighbor has 2 boys and S be the event that the neighbor has a son

The possible options are {BB, BG, GB, GG}

Event S, the neighbor has a son is the set S={BB, BG, GB}

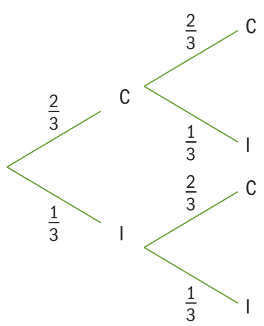
Event T, that the neighbor has two boys is the set T={BB}

We require $P(T|S) = \frac{P(T \cap S)}{P(S)} = \frac{P(\{BB\})}{P(\{BB, BG, GB\})}$

$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

## Exercise 3H

**1** **a**

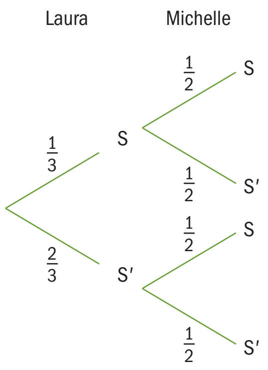


**b** P(C and I) or P(I and C)

$= \left(\frac{2}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{3}\right) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

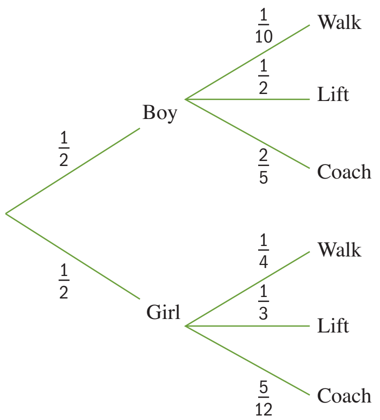
**c** $1 - \text{P(none correct)} = 1 - \left(\frac{1}{3} \times \frac{1}{3}\right) = 1 - \left(\frac{1}{9}\right) = \frac{8}{9}$

**2**




P(neither will score in the next game) $= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

**3**




**a** $\frac{1}{2} \times \frac{5}{12} = \frac{5}{24}$

**b** $\left(\frac{1}{2} \times \frac{2}{5}\right) + \left(\frac{1}{2} \times \frac{5}{12}\right) = \frac{1}{5} + \frac{5}{24} = \frac{49}{120}$

**4** P(Head) $= \frac{2}{3}$ We require HHT or HTH or THH. Each has probability $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$. Therefore

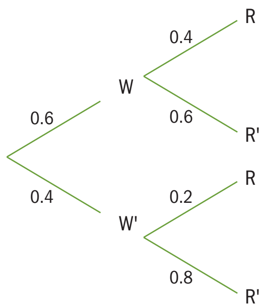
P (HHT or HTH or THH) $= 3 \times \frac{4}{27} = \frac{4}{9}$

**5** **a** P(Prime) = 0.4.

P(exactly one Prime) = P(Prime and not prime) + P(not prime and prime) = (0.4 × 0.6) + (0.6 × 0.4) = 0.48

**b** P(at least one prime) = 1 – P(no primes) = 1 – (0.6 × 0.6) = 1 – 0.36 = 0.64

**6** **a**




**b** P(rainy) = P(W and R) or P(W’ and R) = (0.6 × 0.4) + (0.4 × 0.2) = 0.24 + 0.08 = 0.32

**c** P(two successive days **not** being rainy) = P(not rainy) × P(not rainy)

P(not rainy) = 1 – 0.32 = 0.68

0.68 × 0.68 = 0.4624

## Exercise 3I

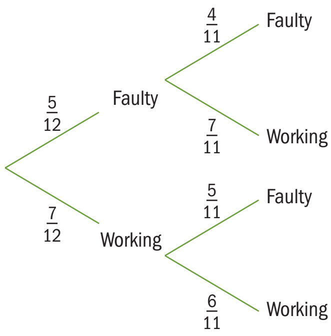
**1** **a** P(picture card) $= \frac{12}{52}$

We require $\frac{12}{52} \times \frac{11}{51} \times \frac{10}{50} = \frac{11}{1105}$

**b** We require $PP\overline{P}$ *or* $P\overline{P}P$ *or* $\overline{P}PP$. Each of these has equal probability of $\frac{12}{52} \times \frac{11}{51} \times \frac{40}{50} = \frac{44}{1105}$

$P(PP\overline{P} \text{ or } P\overline{P}P \text{ or } \overline{P}PP) = 3 \times \frac{44}{1105} = \frac{132}{1105}$

**2**




**a** P(two faulty) $= \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

**b** P(exactly one faulty)

$= \left(\frac{5}{12} \times \frac{7}{11}\right) + \left(\frac{7}{12} \times \frac{5}{11}\right) = \frac{35}{132} + \frac{35}{132} = \frac{35}{66}$

**c** $P(F_2 \mid \text{exactly one faulty pen})$

$= \frac{P(F_2 \text{ and exactly one faulty pen})}{P(\text{exactly one faulty pen})} = \frac{\frac{7}{12} \times \frac{5}{11}}{\frac{35}{66}} = \frac{1}{2}$

**3** **a** $\frac{3}{9} \times \frac{2}{8} = \frac{1}{12}$

**b** P(RR or GG or YY) =

$\left(\frac{4}{9} \times \frac{3}{8}\right) + \left(\frac{3}{9} \times \frac{2}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right) = \frac{5}{18}$

**c** We require P(YY or YG or GG or GY)

$= \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{2}{9} \times \frac{3}{8}\right) + \left(\frac{3}{9} \times \frac{2}{8}\right) + \left(\frac{3}{9} \times \frac{2}{8}\right) = \frac{5}{18}$

**d** We require $P(\overline{R} \text{ and } \overline{R}) = \frac{5}{9} \times \frac{4}{8} = \frac{5}{18}$

**4** P(one of each color) = P(RBOP in any order)

P(RBOP) $= \frac{5}{14} \times \frac{4}{13} \times \frac{3}{12} \times \frac{2}{11} = \frac{5}{1001}$

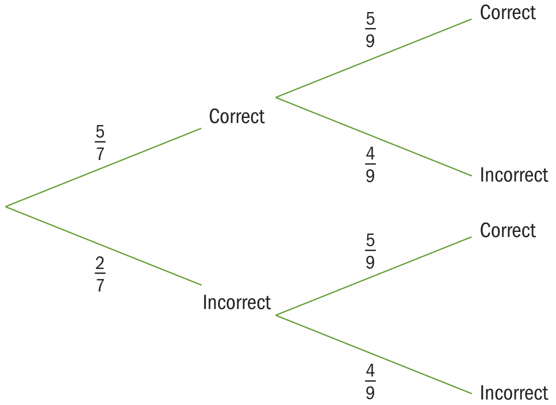
We can arrange RBOP in 24 ways

Therefore required probability $= 24 \times \frac{5}{1001} = \frac{30}{1001}$

**5** **a** $\frac{4}{10} = \frac{2}{5}$

**b** $\left(\frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{6}{10} \times \frac{4}{9}\right) = \frac{8}{15}$

**6**




**a** P(at least one of the students answers the question correctly) = 1 – P(both incorrect)

$= 1 - \left(\frac{2}{7} \times \frac{4}{9}\right) = \frac{55}{63}$

**b** P(Billy correct given that the answer is correct) $= \dfrac{\frac{5}{7}}{\frac{55}{63}} = \frac{9}{11}$

**c** P(Natasha correct given that the answer is correct) $= \dfrac{\frac{5}{9}}{\frac{55}{63}} = \frac{7}{11}$

**d** P(two correct answers given that there were one) $= \dfrac{\frac{5}{7} \times \frac{5}{9}}{\frac{55}{63}} = \frac{25}{55} = \frac{5}{11}$

## Review exercise

**1** There are 90 numbers from 10 to 99 inclusive.

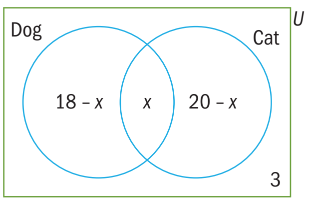
**a** {10, 15, 20,…, 90, 95} or every 5th number is divisible by 5 so $\frac{18}{90} = \frac{1}{5}$

**b** {3, 6, 9, 12, …, 96, 99} or every 3rd number is divisible by 3 so $\frac{1}{3}$

**c** {51, 52, 53, …98, 99} $\frac{49}{90}$

**d** {16, 25, 36, 49, 64, 81} $\frac{6}{90} = \frac{1}{15}$

**2** Let $n(C \cap D) = x$



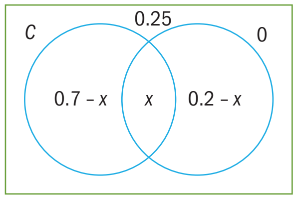

$18 - x + x + 20 - x + 3 = 30$

$41 - x = 30$

$x = 11$

$\text{P}(\text{Cat and Dog}) = \frac{11}{30}$

**3** Let $\text{P}(C \cap D) = x$




**a** $0.7 - x + x + 0.2 - x + 0.25 = 1$

$1.15 - x = 1$

$x = 0.15$

$\therefore P(C \cap D') = P(C) - P(C \cap D)$

$= 0.7 - 0.15 = 0.55$

**b** Not independent since $\text{P}(C \cap D) = 0.15$ and $\text{P}(C) \times \text{P}(D) = 0.7 \times 0.2 = 0.14$

**4 a** We require $\text{P}(A \cap B)$

$\text{P}(A / B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$

$0.1 = \frac{\text{P}(A \cap B)}{0.2}$

$\text{P}(A \cap B) = 0.1 \times 0.2 = 0.02$

**b** We require $\text{P}(A \cup B)$

$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$

$\text{P}(A \cup B) = 0.6 + 0.2 - 0.02$

$\text{P}(A \cap B) = 0.78$

**c** We require

$\text{P}(A \cup B) - \text{P}(A \cap B)$

$= 0.78 - 0.02$

$= 0.76$

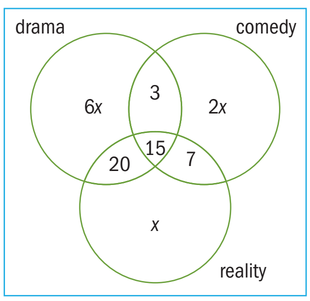
**d** We require $\text{P}(B \mid A)$

$\text{P}(B|A) = \frac{\text{P}(B \cap A)}{\text{P}(A)}$

$= \frac{0.02}{0.6} = \frac{1}{30} = 0.0333$

**5 a** $6x$

**b**




**c** $6x + 3 + 2x + 20 + 15 + 7 + x + 10 = 100$

$9x + 55 = 100$

$9x = 45$

$x = 5$

## Review exercise

**1** **a** $P(C \cap D) = P(C|D) \times P(D)$
$= 0.6 \times 0.5$
$= 0.3$

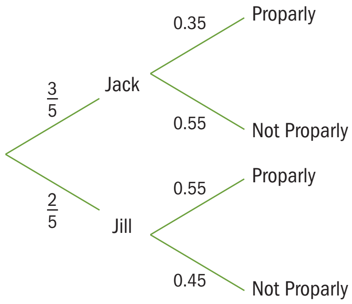
**b** No since $P(C \text{ and } D) \neq 0$

**c** No since $P(C) \times P(D) = 0.4 \times 0.5 = 0.2 \neq P(C \text{ and } D)$

**d** $P(C \cup D) = P(C) + P(D) - P(C \cap D)$
$= 0.4 + 0.5 - 0.3$
$= 0.6$

**e** $P(D|C) = \frac{P(D \cap C)}{P(C)}$
$= \frac{0.3}{0.4} = 0.75$

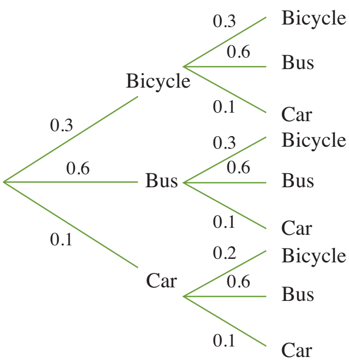
**2**




**a** P(Properly) = P(Jack and Properly) + P(Jill and Properly) = $\left(\frac{3}{5} \times 0.35\right) + \left(\frac{2}{5} \times 0.55\right)$
$= 0.21 + 0.22 = 0.43$

**b** $P(\text{Jill} \mid \text{Not Properly}) = \frac{P(\text{Jill and not properly})}{P(\text{Not Properly})}$
$= \frac{\frac{2}{5} \times 0.45}{0.57} = 0.316$

**3** **a**




**b** **i** P(Travels by bicycle on Monday and Tuesday) = 0.3 × 0.3 = 0.09

**ii** P(Travels by bicycle on Monday and by bus on Tuesday) = 0.3 × 0.6 = 0.18

**iii** P(Travels by the same method of travel on Monday and Tuesday) = (0.3 × 0.3) + (0.6 × 0.6) + (0.1 × 0.1) = 0.46

**c** P(not by bicycle on 3 days) = 0.7 × 0.7 × 0.7 = 0.343

**d** P(twice by car & once by bus)
= P(car ∩ car ∩ bus) + P(car ∩ bus ∩ car) + P(bus ∩ car ∩ car)

Now P(car ∩ car ∩ bus) = (0.1 × 0.1 × 0.6)

So P(twice by car & once by bus)
= 3 × (0.1 × 0.1 × 0.6) = 0.018

P(twice by bicycle & once by car)
= P(bike ∩ bike ∩ car) + P(bike ∩ car ∩ bike) + P(car ∩ bike ∩ bike)

Now P(bike ∩ bike ∩ car) = (0.3 × 0.3 × 0.1)

So P(twice by bicycle & once by car)
= 3 × (0.3 × 0.3 × 0.1) = 0.027

Thus,

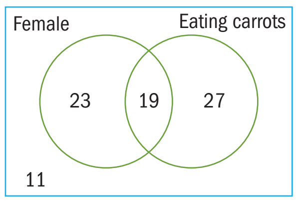
P(twice by car & once by bus or twice by bicycle & once by car) = 0.018 + 0.027 = 0.045

**4** **a** $\frac{6}{16} = \frac{3}{8}$

**b** $\frac{10}{15} = \frac{2}{3}$

**c** $\frac{5}{15} \times \frac{4}{14} = \frac{2}{21}$

**5**




$n$(Female and not eating carrots) = 23
$n$(Female and eating carrots) = 42 − 23 = 19
$n$(not female and eating carrots) = $x$
Now $70 - (19 + x) = 34$
$x = 17.$

**a** P(a rabbit is male and not eating carrots) = $\frac{11}{70}$

**b** P(a rabbit is female | that it is eating carrots) = $\frac{\frac{19}{70}}{\frac{36}{70}} = \frac{19}{36}$

**c** No; $P(F) \times P(C) = \frac{42}{70} \times \frac{36}{70} = \frac{78}{2450} \neq P(F \text{ and } C)$