15 Probability distributions

Answers

Skills check

1 a $\bar{x} = \frac{\sum fx}{\sum f} = \frac{(3\times3)+(4\times5)+(5\times7)+(6\times9)+(7\times6)+(8\times2)}{3+5+7+9+6+2}$

$= \frac{176}{32} = 5.5$

b $\bar{x} = \frac{(10\times3)+(12\times10)+(15\times15)+(17\times9)+(20\times2)}{3+10+15+9+2}$

$= \frac{568}{39} = 14.6$ (3 sf)

2 a $\binom{6}{2} = \frac{6!}{2!4!} = \frac{6\times5}{2} = 15$

b $\binom{8}{5} = \frac{8!}{5!3!} = \frac{8\times7\times6}{3\times2\times1} = 56$

c $\binom{9}{6}(0.3)^3(0.7)^6 = 0.267$

3 a $\frac{5.5}{x} = 3.2 \quad \therefore 5.5 = 3.2x \quad \therefore x = \frac{5.5}{3.2} = 1.71875$

b $\frac{x-2.5}{1.2} = 0.4 \quad \therefore x - 2.5 = 0.48 \quad \therefore x = 2.98$

c $\frac{9-x}{0.2} = 1.6 \quad \therefore 9 - x = 0.32 \therefore x = 9 - 0.32 = 8.68$

Exercise 15A

1 a discrete

b continuous

c continuous

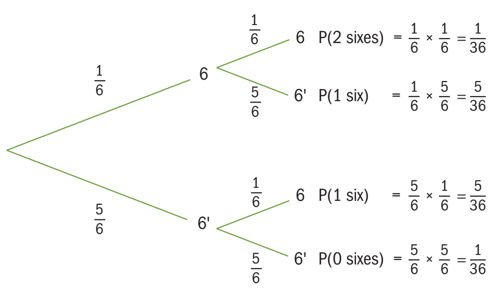
b discrete

2 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b



$P(2 \text{ sixes}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(1 \text{ six}) = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

$P(1 \text{ six}) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$P(0 \text{ sixes}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

x	0	1	2
$P(X = x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

c

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

x	1	2	3	4	5	6
$P(X = x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

d

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	8	9	12	15	18
4	4	10	12	16	20	24
5	5	12	15	20	25	30
6	6	14	18	24	30	36

x	1	2	3	4	5	6	8	9	10
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$
x	12	15	16	18	20	24	25	30	36
$P(X = x)$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

3 a The faces are numbered 1, 2, 2, 3, 3, 3

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

t	2	3	4	5	6
P(*T* = *t*)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{12}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

b $P(T > 4) = \frac{12}{36} + \frac{9}{36} = \frac{21}{36} = \frac{7}{12}$

4 a

no. on dice	1	2	3	4	5	6
s	2	1	6	2	10	3

s	1	2	3	6	10
P(*S* = *s*)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b $P(S > 2) = \frac{3}{6} = \frac{1}{2}$

5 a $\frac{1}{3} + \frac{1}{3} + c + c = 1$

$2c = \frac{1}{3} \quad \therefore\ c = \frac{1}{6}$

b $P(1 < X < 4) = P(X = 2) + P(X = 3) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

6 $P(Y = y) = cy^3 \quad y = 1, 2, 3$

y	1	2	3
P(*Y* = *y*)	*c*	8*c*	27*c*

$36c = 1 \quad \therefore\ c = \frac{1}{36}$

7 $2k + 4k^2 + 6k^2 + k = 1$

$10k^2 + 3k - 1 = 0$

$(5k - 1)(2k + 1) = 0$

$k = \frac{1}{5}$ (k cannot be negative)

8 $P(X = x) = k\left(\frac{1}{3}\right)^{x-1} \quad x = 1, 2, 3, 4$

x	1	2	3	4
P(*X* = *x*)	k	$\frac{1}{3}k$	$\frac{1}{9}k$	$\frac{1}{27}k$

$k + \frac{1}{3}k + \frac{1}{9}k + \frac{1}{27}k = 1$

$\frac{40}{27}k = 1 \quad \therefore\ k = \frac{27}{40}$

9 a

x	0	1	2	3	4	5
P(*X* = *x*)	a	a	a	b	b	b

$3a + 3b = 1 \quad (1) \qquad P(X \geq 2) = 3P(X < 2)$

$a + 3b = 6a$

$3b = 5a \quad (2)$

substitute (2) and (1) $\quad 3a + 5a = 1$

$a = \frac{1}{8} \qquad b = \frac{5}{24}$

b $P(5, 3) = \frac{5}{24} \times \frac{5}{24} = \frac{25}{576} \quad P(3, 5) = \frac{25}{576}$

$P(5, 4) = \frac{25}{576} \quad P(4, 5) = \frac{25}{576} \quad P(5, 5) = \frac{25}{576}$

$P(\text{sum} > 7) = \frac{25}{576}$

10 a $P(C = 3) = P(A = 1 \text{ and } B = 2) + P(A = 2 \text{ and } B = 1)$

$= \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} + \frac{1}{6} = \frac{2}{9} + \frac{1}{18} = \frac{5}{18}$

b $P(C = 2) = P(A = 1 \text{ and } B = 1) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

$P(C = 4) = P(A = 1 \text{ and } B = 3) + P(A = 2 \text{ and } B = 2) + P(A = 3 \text{ and } B = 1)$

$P(C = 5) = P(A = 2 \text{ and } B = 3) + P(A = 3 \text{ and } B = 2)$

$\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{3} = \frac{5}{18}$

$P(C = 6) = P(A = 3 \text{ and } B = 3) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

c	2	3	4	5	6
P(*C* = *c*)	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{6}{18}$	$\frac{5}{18}$	$\frac{1}{18}$

Investigation – dice scores

1

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

d	0	1	2	3	4	5
P(*D* = *d*)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

2

d	0	1	2	3	4	5
Expected frequency	6	10	8	6	4	2

3 Mean $= \frac{(0 \times 6) + (1 \times 10) + (2 \times 8) + (3 \times 6) + (4 \times 4) + (5 \times 2)}{36}$

$= \frac{70}{36} = \frac{35}{18}$

4

d	0	1	2	3	4	5
Expected frequency	$\frac{150}{9}$	$\frac{250}{9}$	$\frac{200}{9}$	$\frac{150}{9}$	$\frac{100}{9}$	$\frac{50}{9}$

Mean =

$$\frac{\left(0 \times \frac{150}{9}\right) + \left(1 \times \frac{250}{9}\right) + \left(2 \times \frac{200}{9}\right) + \left(3 \times \frac{150}{9}\right) + \left(4 \times \frac{100}{9}\right) + \left(5 \times \frac{50}{9}\right)}{100}$$

$= \frac{35}{18}$

5 The means are the same

6 $\frac{35}{18}$

Exercise 15B

1

x	1	4	9	16	24	36
P(*X* = *x*)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(X) = \left(1\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(9\times\frac{1}{6}\right)+\left(16\times\frac{1}{6}\right)+\left(25\times\frac{1}{6}\right)+\left(36\times\frac{1}{6}\right)$

$= \frac{91}{6} = 15.2$ (3 sf)

2 $\frac{3}{6} + x + y = 1 \quad \therefore x + y = \frac{1}{2} \quad (1)$

$E(Z) = 5\frac{2}{3} \quad \therefore$

$\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+7x+11y=\frac{17}{3}$

$7x + 11y = 4 \quad (2)$

Solving (1) and (2), $x = \frac{3}{8}, \quad y = \frac{1}{8}$

3

x	1	2	3	5	8	13
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(X) = \frac{1}{6}(1 + 2 + 3 + 5 + 8 + 13) = \frac{16}{3}$ or $5\frac{1}{3}$

4

x	1	2	3	4	5	6	7	8
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$\frac{8}{36}$

$E(X) = \frac{1}{36}+\frac{4}{36}+\frac{9}{36}+\frac{16}{36}+\frac{25}{36}+\frac{36}{36}+\frac{49}{36}+\frac{64}{36}$

$= \frac{204}{36} = 5\frac{2}{3}$

5 **a**

x	1	2	3	4	5	6	7	8	9
P(X = x)	k	2k	3k	4k	5k	4k	3k	2k	k

$25k = 1 \quad \therefore k = \frac{1}{25}$

b From symmetry, $E(X) = 5$

6 **a**

x	1	2	3
P(X = x)	0.2	1 − k	k − 0.2

b $0 \le 1 - k \le 1$ and $0 \le k - 0.2 \le 1$

$-1 \le -k \le 0 \qquad 0.2 \le k \le 1.2$

$1 \ge k \ge 0$

$\therefore 0.2 \le k \le 1$

c Mean $= 0.2 + 2(1 - k) + 3(k - 0.2)$

$= 0.2 + 2 - 2k + 3k - 0.6 = k + 1.6$

7

x	1	2	4
P(x = x)	a	0.3	6

$a + b = 0.7 \quad (1)$

mean $= 2.8 \qquad \therefore a + 0.6 + 46 = 28$

$a + 4b = 2.2 \quad (2)$

solving (1) and (2), $a = 0.2, b = 0.5$

$\therefore P(x = 1) = 0.2$

8 **a** $P(R = 1) = \frac{2}{10} = \frac{1}{5}$

$P(R = 2) = \frac{8}{10}\times\frac{2}{9} = \frac{16}{90} = \frac{8}{45}$

$P(R = 3) = \frac{8}{10}\times\frac{7}{9}\times\frac{2}{8} = \frac{14}{90} = \frac{7}{45}$

$P(R = 4) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{2}{7} = \frac{12}{90} = \frac{6}{45}$

$P(R = 5) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{2}{6} = \frac{10}{90} = \frac{5}{45}$

$P(R = 6) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}\times\frac{2}{5} = \frac{8}{90} = \frac{4}{45}$

$P(R = 7) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4} = \frac{6}{90} = \frac{3}{45}$

$P(R = 8) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3} = \frac{4}{90} = \frac{2}{45}$

$P(R = 9) = \frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}$

$= \frac{2}{90} = \frac{1}{45}$

r	1	2	3	4	5	6	7	8	9
P(R = r)	$\frac{9}{45}$	$\frac{8}{45}$	$\frac{7}{45}$	$\frac{6}{45}$	$\frac{5}{45}$	$\frac{4}{45}$	$\frac{3}{45}$	$\frac{2}{45}$	$\frac{1}{45}$

b mean $= \frac{1}{45}(9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9) = 3\frac{2}{3}$

9 **a** $P(R = 2) = \frac{8}{10}\times\frac{2}{10} = \frac{16}{100} = \frac{4}{25}$

b $P(R = 3) = \left(\frac{8}{10}\right)^2 \times \frac{2}{10} = \frac{16}{125}$

c $P(R = n) = 0.8^{n-1} \times 0.2$

d 1

10 **a** $P(Z = 0) + 0.2 + 0.05 + 0.001 + 0.0001 = 1$

$P(Z = 0) = 1 - 0.2511$

$= 0.7489$

b $E(Z) = 0 + 0.4 + 1 + 0.2 + 0.1$

$= 1.7$

\$1.70 is the expected winnings on a ticket

c A ticket costs \$2, but you only expect to win \$1.70. Therefore you expect to lose \$0.30

Investigation – the binomial quiz

You would expect to get 2.5 questions right

$P(3 \text{ right}) = \binom{5}{3}(0.5)^3(0.5)^2 = 0.3125$

Exercise 15C

1 $X \sim B\left(4, \frac{1}{2}\right)$

a $P(X = 1) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{1}{4}$

b $P(X < 1) = P(X = 0) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

c $P(X \le 1) = P(X = 0 \text{ or } 1) = \frac{1}{16}+\frac{1}{4}=\frac{5}{16}$

d $P(X \ge 1) = 1 - P(X = 0) = 1 - \frac{1}{16} = \frac{15}{16}$

2 using a GDC:

a 0.329 **b** 0.351

c 0.680 **d** 0.649

3 using a GDC:

a $P(X = 5) = 0.0389$

b $P(X < 5) = 0.952$

c $P(X > 5) = 0.00870$

d $P(X \geq 1) = 0.932$

Exercise 15 D

(Using a GDC where possible)

1 $X \sim B(4, 0.25)$

x	0	1	2	3	4
P(X= x)	0.316	0.422	0.211	0.469	0.00391

most likely outcome is 1 red face with probability 0.422

2 $X \sim B(8, 0.55)$

a $P(X= 5) = 0.257$

b $P(\text{at least 5 times}) = P(X \leq 3) = 0.260$

3 $X \sim B(16, 0.55)$

a $P(X = 0) = 0.851$

b $P(\text{13 not faulty}) = P(X = 3) = 0.000491$

c $P(X \geq 2) = 0.0109$

4 $X \sim B(10, 0.25)$

a $P(X = 5) = 0.0584$

b $P(\text{at least 3 free}) = P(X \leq 7) = 0.9996$

5 $X \sim B(5, 0.24)$

$P(X \leq 3) = 0.913$

6 $X \sim B(6, 0.15)$

a $P(X > 1) = 0.224$

b $P(X = 1) = 0.399$

7 $X \sim B(15, 0.05)$

a **i** $P(X = 3) = 0.0307$

ii $P(X = 0) = 0.463$

iii $P(X \geq 2) = 0.171$

b **i** $(0.46329...)^2 = 0.215$

ii $0.17095....)^2 = 0.0292$

iii $0.46329..... \times 0.17095.... \times 2 = 0.158$

Exercise 15 E

1 $X \sim B(n, 0.6)$ $\quad P(X < 1) = 0.0256$

$P(X = 0) = 0.0256$

$(0.4)^n = 0.0256$

$n \log 0.4 = \log 0.0256$

$n = 4$

2 $X \sim B(n, 0.01)$ $\quad P(x = 0) > 0.5$

$0.99^n > 0.5$

$n \log 0.99 > \log 0.5$

$n < \frac{\log 0.5}{\log 0.99} = 68.9$, so the largest sample size is 68

3 $X \sim B(n, 0.2)$ $\quad P(X \geq 1) > 0.75$

$1 - P(X = 0) > 0.75$

$1 - 0.8^n > 0.75$

$0.25 > 0.8^n$

$0.8^n < 0.25$

$n \log 0.8 < \log 0.25$

$n > \frac{\log 0.25}{\log 0.8}$

$n > 6.21$

$\therefore$ least value of $n = 7$

4 $X \sim B(n, 0.3)$ $\quad P(x \geq 1) > 0.95$

$1 - P(x = 0) > 0.95$

$1 - 0.7^n > 0.95$

$0.05 > 0.7^n$

$0.7^n < 0.05$

$n \log 0.7 < \log 0.05$

$n > \frac{\log 0.05}{\log 0.7}$

$n > 8.399$

$\therefore$ least number of attempts is 9

5 $X \sim B(n, 0.5)$ $\quad P(X \geq 1) > 0.99$

$1 - P(X = 0) \geq 0.99$

$1 - 0.5^n \geq 0.99$

$0.01 \geq 0.5^n$

$0.5^n \leq 0.01$

$n \log 0.5 \leq \log 0.01$

$n \geq \frac{\log 0.01}{\log 0.5}$

$n > 6.64$ so the coin must be tossed 7 times.

Exercise 15F

1 **a** $X \sim B(40, 0.5)$ $\quad E(X) = 40 \times 0.5 = 20$

b $X \sim B\left(40, \frac{1}{6}\right)$ $\quad E(X) = 40 \times \frac{1}{6} = 6\frac{2}{3}$

c $X \sim B(40, 0.25)$ $\quad E(X) = 40 \times 0.25 = 10$

2 $X \sim B(n, P)$ $\quad$ mean $= 10$ $\quad P = 0.4$ $\quad nP = 10$

$n \times 0.4 = 10$ $\quad \therefore n = 25$

3 **a** $X \sim B(15, 0.25)$

b mean $= 15 \times 0.25 = 3.75$

c $P(X \geq 10) = 0.000795$

4 **a** total number of girls =
$(1 \times 34) + (2 \times 40) + (3 \times 13) = 158$

total number of children $= 100 \times 3 = 300$

$\therefore P(\text{girl}) = \frac{153}{300} = 0.51$

b $X \sim B(3, 0.51)$ $\quad P(x = 2) = 0.382$

expected number of families $= 0.382 \times 100$
$= 38.2$

Exercise 15G

1 $X \sim B\left(0, \frac{1}{4}\right)$

Mean $= 0 \times \frac{1}{4} = 0$

Variance $= 0 \times \frac{1}{4} \times \left(1 - \frac{1}{4}\right) = 0$

2 B(12, 0.6)

Mean $= 12 \times 0.6 = 7.2$

Variance $= 12 \times 0.6 \times 0.4 = 2.88$

Standard deviation $= \sqrt{2.88} = 1.70$ (3 sf)

3 $X \sim B\left(40, \frac{1}{2}\right)$

Mean $= 40 \times \frac{1}{2} = 20$

Variance $= 40 \times \frac{1}{2} \times \frac{1}{2} = 10$

Standard deviation $= \sqrt{10} = 3.16$ (3 sf)

4 $X \sim B\left(10, \frac{1}{6}\right)$

a $E(X) = 10 \times \frac{1}{6} = \frac{5}{3}$

b $Var(X) = 10 \times \frac{1}{6} \times \frac{5}{6} = \frac{25}{18}$

c $P(X < \mu) = P\left(X < \frac{5}{3}\right) = 0.485$ (3 sf) (using binomial CDF on the GDC)

5 $X \sim B\left(22, \frac{1}{5}\right)$

a $E(X) = 22 \times \frac{1}{5} = \frac{22}{5}$

b $Var(X) = 22 \times \frac{1}{5} \times \frac{4}{5} = \frac{88}{25}$

c $P(X < 4) = 0.332$ (3 sf) using binomial CDF on the GDC)

6 $X \sim B(n, p)$

$E(X) = 4.5$, $Var(X) = 3.15$

$E(X) = np = 4.5$

$$Var(X) = npq = np(1 - p) = 3.15$$
$$4.5(1 - p) = 3.15$$
$$1 - p = \frac{3.15}{4.5}$$
$$= 0.7$$
$$p = 1 - 0.7$$
$$= 0.3$$

$np = 4.5$

$n = \frac{4.5}{p} = \frac{4.5}{0.3} = 15$

$P(X \geq 3) = 1 - P(X < 3) = 1 - 0.126828$

$= 0.873$ (3 sf)

7 $X \sim B(n, p)$

$E(X) = 7.8$

$p = 0.3$

a $E(X) = np = 7.8$

$0.3n = 7.8$

$n = \frac{7.8}{0.3}$

$n = 26$

b $Var(X) = npq = np(1 - p)$

$= 26 \times 0.3 \times 0.7$

$= 5.46$

8 $X \sim B(n, p)$

$E(X) = 9.6$

$Var(X) = 1.92$

$E(X) = np = 9.6$

$$Var(X) = npq = np(1 - p) = 1.92$$
$$9.6(1 - p) = 1.92$$
$$9.6 - 9.6p = 1.92$$
$$9.6p = 7.68$$
$$p = 0.8$$
$$n = \frac{9.6}{0.8}$$
$$n = 12$$

$P(X = 6) = 0.0155$ (using binomial PDF on the GDC)

Exercise 15H

Using GDC:

1 **a** 0.683

b 0.954

c 0.997

2 **a** $P(1 < Z < 2) + p(-2 < Z < -1) = 0.272$

b $P(0.5 < Z < 1.5) + p(-1.5 < Z < -0.5) = 0.483$

3 **a** $P(Z > 1) = 0.159$

b $P(Z > 2.4) = 0.00820$

4 **a** $P(Z > -1) = 0.159$

b $P(Z < -1.75) = 0.0401$

5 **a** 0.742

b 0.236

c 0.0359

d 0.977

e 0.390

6 **a** 0.306

b 0.595

c 0.285

7 **a** $P(|Z| < 0.4) = P(-0.4 < Z < 0.4) = 0.311$

b $P(|Z| > 1.24) = 1 - P(1 Z 1 < 1.24)$

$= 1 - P(-1.24 < Z < 1.24)$

$= 0.215$

Exercise 15I

Using GDC:

1 **a** 0.655

b 0.841

c 0.186

d 0.5

2 **a** 0.672

b 0.748

c 0.345

3 **a** 0.994

b 0.997

c 0.494

Exercise 15J

1 $X \sim N(100, 20^2)$

a $P(X < 130) = 0.933$

b $P(X > 90) = 0.691$

c $P(80 < X < 125) = 0.736$

2 $X \sim N(4, 0.25^2)$

$P(3.5 < X < 4.5) = 0.9545$

number of acceptable bolts $= 0.9545 \times 500 = 477$

3 $X \sim N(14, 4^2)$

a $P(X > 20) = 0.0668$

b $P(X > 10) = 0.159 = 15.9\%$

4 $X \sim N(551.3, 15^2)$

$P(X > 550) = 0.535 = 53.5\%$

5 $X \sim N(500, 20^2)$

a $P(X < 475) = 0.106$

b $(0.1056\ \ldots)^3 = 0.00118$

Exercise 15K

1 **a** $P(Z > a) = 0.922,\ a = 1.42$

b $P(Z > a) = 0.342$

$\therefore P(Z < a) = 0.658,\ a = 0.407$

c $P(Z > a) = 0.005 \quad \therefore P(Z < a) = 0.995,$

$a = 2.58$

2 **a** $P(1 < Z < a) = 0.12$

$P(Z < 1) = 0.8413 \quad \therefore P(Z < a) = 0.9613$

$\therefore a = 1.77$

b $P(a < Z < 1.6) = 0.787$

$P(Z > 1.6) = 0.0548$

$\therefore P(Z < a) = 1 - (0.787 + 0.0548)$

$= 0.1582$

$\therefore a = -1.00$

c $P(a < Z < -0.3) = 0.182$

$P(Z > -0.3) = 0.6179$

$\therefore P(Z < a) = 1 - (0.182 + 0.6179) = 0.2001$

$\therefore a = -0.841$

3 **a** $P(-a < Z < a) = 0.3$

$\therefore P(Z < a) = 0.65 \qquad \therefore a = 0.385$

b $P(1\ Z\ 1 > a) = 0.1096$

$\therefore P(Z < a) = 0.9452 \qquad \therefore a = 1.60$

4 **a** $P(Z < Z) = 0.95 \quad \therefore Z = 1.64$

b $P(Z < Z) = 0.8 \quad \therefore Z = 0.842$

Exercise 15L

1 $X \sim N(5.5, 0.2^2) \quad P(X > a) = 0.235$

$P(x < a) = 0.765 \quad \therefore a = 5.64$

2 $M \sim N(420, 10^2)$

a $P(M < a) = 0.25 \quad \therefore a = 413$

b $P(M < b) = 0.9 \quad \therefore b = 433$

3 $X \sim N(502, 1.6^2)$

a $P(x < 500) = 0.106$

b $P(500 < x < 505) = 0.864$ or 86.4%

c $P(x < b) = 0.975 \quad b = 505.1 \quad a = 498.9$

$a = 499 \quad b = 505$

4 $X \sim N(550, 25^2)$

a $P(520 < X < 500) = 0.673$

b $P(X > a) = 01 \quad \therefore P(X < a) = 0.9 \quad \therefore a = 582a$

Exercise 15M

1 $X \sim N(30, \sigma^2) \quad P(X > 40) = 0.115$

$Z = \frac{40-30}{\sigma} = \frac{10}{\sigma}$

$\therefore \frac{10}{\sigma} = 1.2004 \quad \therefore \sigma = 8.33$

2 $X \sim N(\mu, 4^2) \quad P(X < 20.5) = 0.9$

$Z = \frac{20.5-\mu}{4} \quad \therefore P\left(Z < \frac{20.5-\mu}{4}\right) = 0.9$

$\therefore \frac{20.5-\mu}{4} = 1.28155$

$\therefore \mu = 15.4$

3 $X \sim N(\mu, \sigma^2) \quad p(X > 58.39) = 0.0217$

$P(X < 41.82) = 0.0287$

$Z = \frac{59.39-\mu}{\sigma} \qquad Z = \frac{41.82-\mu}{\sigma}$

$P\left(Z > \frac{58.39-\mu}{\sigma}\right) = 0.0217$

$\therefore P\left(Z < \frac{58.39-\mu}{\sigma}\right) = 0.9783 \quad \therefore \frac{58.39-\mu}{\sigma} = 2.0198$

$P\left(Z < \frac{41.82-\mu}{\sigma}\right) = 0.0287 \quad \therefore \frac{41.82-\mu}{\sigma} = -1.9003$

$\therefore \mu = 49.9 \qquad \sigma = 4.23$

4 $X \sim N(\mu, \sigma^2) \quad P(X < 89) = 0.90 \quad P(X < 94) = 0.95$

$Z = \frac{89-\mu}{\sigma} \qquad Z = \frac{94-\mu}{\sigma}$

$P\left(Z < \frac{89-\mu}{\sigma}\right) = 0.90 \quad \therefore \frac{89-\mu}{\sigma} = 1.28155$

$P\left(Z < \frac{94-\mu}{\sigma}\right) = 0.95 \quad \therefore \frac{94-\mu}{\sigma} = 1.64485$

$\therefore \mu = 71.4 \qquad \sigma = 13.8$

5 $X \sim N(136, \sigma^2) \quad P(X > 145) = 0.12$

$Z = \frac{145-130}{\sigma} = \frac{9}{\sigma}$

$P\left(Z > \frac{9}{\sigma}\right) = 0.12 \quad \therefore P\left(Z < \frac{9}{\sigma}\right) = 0.88$

$\therefore \frac{9}{\sigma} = 1.175 \qquad \therefore \sigma = 7.66$ cm

6 $X \sim \mathrm{N}(\mu, 20^2)$ $\quad \mathrm{P}(X < 500) = 0.01$

$Z = \frac{500 - \mu}{20}$

$\mathrm{P}\left(Z < \frac{200 - \mu}{20}\right) = 0.01 \quad \therefore \quad \frac{500 - \mu}{20} = -2.326$

$\therefore\ \mu = 546.5$ or 547 g

7 $X \sim \mathrm{N}(0.85, \sigma^2)$ $\quad \mathrm{P}(X < 1.1) = 0.74$

a $Z = \frac{1.1 - 0.85}{\sigma} = \frac{0.25}{\sigma}$

$\mathrm{P}\left(Z < \frac{0.25}{\sigma}\right) = 0.74 \quad \therefore\ \frac{0.25}{\sigma} = 0.6433$

$\therefore\ \sigma = 0.389$ kg

b $\mathrm{P}(x > 1) = 0.350 = 35.0\%$

8 $X \sim \mathrm{N}(\mu, 7^2)$ $\quad \mathrm{P}(X > 68) = 0.025$

$Z = \frac{68 - \mu}{7}$

$\mathrm{P}\left(Z > \frac{68 - \mu}{7}\right) = 0.025 \quad \therefore\ \mathrm{P}\left(Z < \frac{68 - \mu}{7}\right) = 0.975$

$\therefore\ \frac{68 - \mu}{7} = 1.95996 \qquad \therefore\ \mu = 54.3$ cm

9 $X \sim \mathrm{N}(2.9, \sigma^2)$ $\quad \mathrm{P}(X > 3) = 0.35$

$Z = \frac{3 - 2.9}{5} = \frac{0.1}{\sigma}$

$\mathrm{P}\left(Z > \frac{0.1}{\sigma}\right) = 0.35 \quad \therefore\ \mathrm{P}\left(Z < \frac{0.1}{\sigma}\right) = 0.65$

$\therefore\ \frac{0.1}{\sigma} = 0.3853 \qquad \therefore\ \sigma = 0.260$ m

10 **a** $X \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(X < 10.8) = 0.3$ $\mathrm{P}(X > 154) = 0.2$

$Z = \frac{108 - \mu}{\sigma} \qquad Z = \frac{154 - \mu}{\sigma}$

$\mathrm{P}\left(Z < \frac{108 - \mu}{\sigma}\right) = 0.3 \quad \therefore\ \frac{108 - \mu}{\sigma} = -0.5244$

$\mathrm{P}\left(Z > \frac{154 - \mu}{\sigma}\right) = 0.2 \quad \therefore\ \mathrm{P}\left(Z < \frac{154 - \mu}{\sigma}\right) = 0.8$

$\therefore\ \frac{154 - \mu}{\sigma} = 0.8416$

$\therefore\ \mu = 125.7 \qquad \sigma = 33.67$

$\mu = 126 \qquad \sigma = 33.7$

b $\mathrm{P}(X > 117) = 0.605 = 60.5\%$

yes, this is consistent with the normal distribution

11 $X \sim \mathrm{N}(\mu, \sigma^2)$ $\quad \mathrm{P}(X > 495) = 0.95$
$\mathrm{P}(X > 490) = 0.99$

$\frac{495 - \mu}{\sigma} \qquad \frac{490 - \mu}{\sigma}$

$\mathrm{P}\left(Z > \frac{495 - \mu}{\sigma}\right) = 0.95 \quad \therefore\ \mathrm{P}\left(Z < \frac{495 - \mu}{\sigma}\right) = 0.05$

$\therefore\ \frac{495 - \mu}{\sigma} = -1.64485$

$\mathrm{P}\left(Z > \frac{490 - \mu}{\sigma}\right) = 0.99 \quad \therefore\ \mathrm{P}\left(Z < \frac{490 - \mu}{\sigma}\right) = 0.01$

$\therefore\ \frac{490 - \mu}{\sigma} = -2.32635$

Review exercise

1 **a** $0.3 + \frac{1}{k} + \frac{2}{k} + 0.1 + 2.1 = 1$

$\frac{3}{k} = 0.5 \quad \therefore\ k = 6$

b $\mathrm{E}(X) = (-2 \times 0.3) + \left(-1 \times \frac{1}{6}\right) + (1 \times 0.1)$
$+ (2 \times 0.1)$

$= -\frac{7}{15}$

2 **a**

x	1	2	3	4	5
P(X = x)	5c	8c	9c	8c	5c

$35c = 1$

$\therefore c = \frac{1}{35}$

b from symmetry, $\mathrm{E}(X) = 3$

3 $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + x = 1 \quad \therefore\ x = \frac{3}{8}$

$p(6) = p(2, 4) + p(3, 3) + p(4, 2)$

$= \frac{1}{4} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{1}{4} = \frac{13}{64}$

4 **a**

	2	2	4	4
1	2	2	4	4
2	4	4	8	8
3	6	6	12	12
4	8	8	16	16

possible values of P are 2, 4, 6, 8, 12, 16

b From a:

x	2	4	6	8	12	16
P(X = x)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

c $\mathrm{E}(\mathrm{P}) = \frac{2}{8} + \frac{8}{8} + \frac{6}{8} + \frac{16}{8} + \frac{12}{8} + \frac{6}{8}$

$= 7.5$

d

x	£10	£5
P(X = x)	$\frac{1}{4}$	$\frac{3}{4}$

$\mathrm{E}(X) = 10 \times \frac{1}{4} + 5 \times \frac{3}{4} = 6.25$

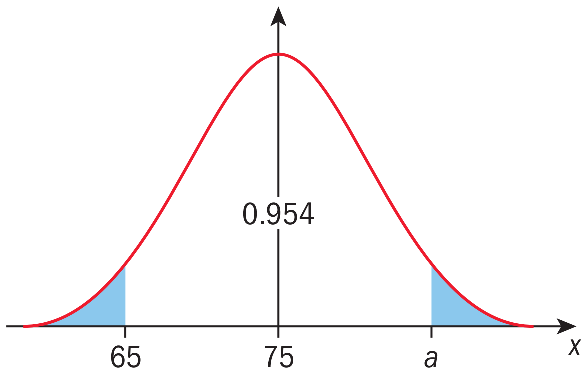
After 10 weeks, expected total $= 6.25 \times 10$
$= £62.50$

5 $X \sim \mathrm{B}\left(5, \frac{1}{3}\right) \qquad \mathrm{P}(X = 3) = \binom{5}{3} \times \left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

$= 10 \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$

6 $X \sim B(2, 0.1)$ $\quad E(X) = nP = 2 \times 0.1 = 0.2$

7 **a**



$P(X < 65) = P(X > a)$

From symmetry, $a = 85$

b $P(X > a) = \frac{1-0.954}{2} = 0.023$

Review exercise

1 **a** $X \sim B\left(3, \frac{1}{3}\right)$ $\quad P(X \geq 1) = \frac{19}{27}$

b

x	-5	1
$P(X = x)$	$\frac{8}{27}$	$\frac{19}{27}$

c **i** $E(X) = -5 \times \frac{8}{27} + 1 \times \frac{19}{27} = -\$\frac{7}{9}$ or $-\$0.78$

$\therefore$ lose \$0.78 (or $\$\frac{7}{9}$)

ii $\frac{7}{9} \times 9 = 7$ $\quad \therefore$ lose \$7

2 $X \sim B(8, 0.3)$

a $P(X = 3) = 0.254$

b $P(X \geq 3) = 0.448$

3 $X \sim B\left(6, \frac{1}{6}\right)$ $\quad P(X = 3) = 0.05358$

$Y \sim B(5, 0.05358)$ $\quad P(Y = 2) = 0.0243$

4 **a** $X \sim B(10, 0.2)$

i $P(X = 4) = 0.881$

ii $P(X > 5) = 0.00637$

b $P(X = 0) = 0.107$ $\quad P(X = 1) = 0.268$

$P(X = 2) = 0.302$

$P(X = 3) = 0.201$ $\quad$ the probabilities continue to decrease after this

$\therefore$ most likely number is 2

c $X \sim B(n, 0.2)$ $\quad P(X \geq 1) > 0.95$

$1 - P(X = 0) > 0.95$

$0.05 > P(X = 0)$

$P(X = 0) < 0.05$

$(0.8)^n < 0.05$

$n \log 0.8 < \log 0.05$

$n > \frac{\log 0.05}{\log 0.8}$

$n > 13.4$

$\therefore$ need 14 points in this sample

5 $P(|Z| \leq a) = 0.85$

$P(-a \leq Z \leq a) = 0.85$

$\therefore P(Z \leq a) = 0.925$ $\quad \therefore a = 1.44$

6 **a** $X \sim N(71, \sigma^2)$ $\quad p(x < 80) = 0.85$

$Z = \frac{80-71}{\sigma} = \frac{9}{\sigma}$

$P\left(Z < \frac{9}{\sigma}\right) = 0.85$ $\quad \therefore \frac{9}{\sigma} = 1.0364$ $\quad \therefore \sigma = 8.68$

b $P(X > 65) = 0.755$

7 $X \sim N(\mu, \sigma^2)$ $\quad P(X < 30) = 0.15$ $\quad P(X > 50) = 0.1$

$Z = \frac{30-\mu}{\sigma}$ $\quad Z = \frac{50-\mu}{\sigma}$

$P\left(Z < \frac{30-\mu}{\sigma}\right) = 0.15$ $\quad \therefore \frac{30-\mu}{\sigma} = -1.03643$

$P\left(Z > \frac{50-\mu}{\sigma}\right) = 0.1$ $\quad \therefore P\left(Z > \frac{50-\mu}{\sigma}\right) = 0.9$

$\therefore \frac{50-\mu}{\sigma} = 1.28155$

$\mu = 38.9$ hours $\quad \sigma = 8.63$ hours

8 **a** $X \sim N(\mu, 2)$ $\quad P(x > 35) = 0.2$

$Z = \frac{35-\mu}{2}$

$P\left(Z > \frac{35-\mu}{2}\right) = 0.2$ $\quad \therefore P\left(Z < \frac{35-\mu}{2}\right) = 0.8$

$\therefore \frac{35-\mu}{2} = 0.8416$

$\therefore \mu = 33.3$

b $X \sim P(5, 0.2)$ $\quad P(X = 0) = 0.328$

c $P(X \geq 2) = 0.263$