

# 10

## Bivariate analysis

### Answers

#### Skills check

Evaluate

1  $2^5 = 32$

2  $3^3 = 27$

3  $7^3 = 343$

4  $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$

5  $\left(\frac{3}{4}\right)^4 = \frac{81}{256}$

6  $0.001^3 = 0.000\ 000\ 001$

State the value of  $n$  in the following equations

1  $2^n = 16$        $n = 4$

2  $3^n = 243$        $n = 5$

3  $7^n = 343$        $n = 3$

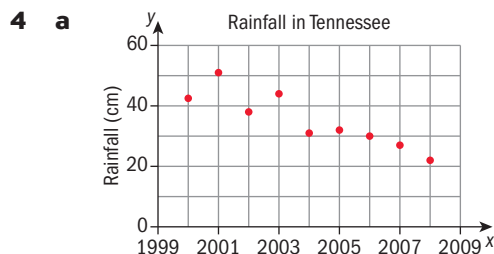
4  $5^n = 625$        $n = 4$

5  $(-4)^n = -64$        $n = 3$

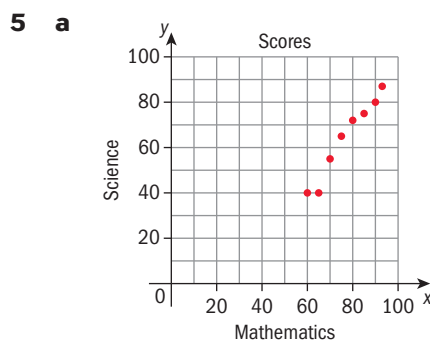
6  $\left(\frac{1}{2}\right)^n = \frac{1}{8}$        $n = 3$

#### Exercise 10A

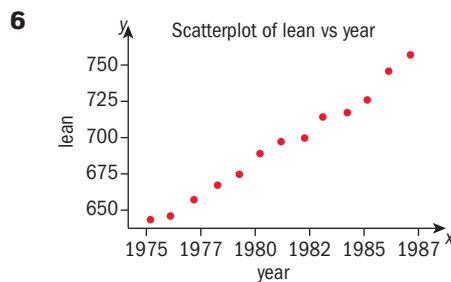
- 1 i a Positive    b Strong  
 ii a Negative    b Weak  
 iii a Negative    b Strong  
 iv a Positive    b Weak  
 v No correlation
- 2 i a positive,    b linear,    c strong,  
 ii a negative,    b linear,    c strong,  
 iii a positive,    b linear,    c Moderate.  
 iv a No association,    b Non linear,  
     c zero.  
 v a positive,    b linear,      c weak.  
 vi a Negative,    b non linear,    c strong.
- 3 a If the independent and dependent variables show a positive correlation then as the independent variable increases the dependent variable increases.  
 b If the independent and dependent variables show a negative correlation then as the independent variable increases the dependent variable decreases.



- b Strong, negative.  
 c As the years increase the rainfall decreases



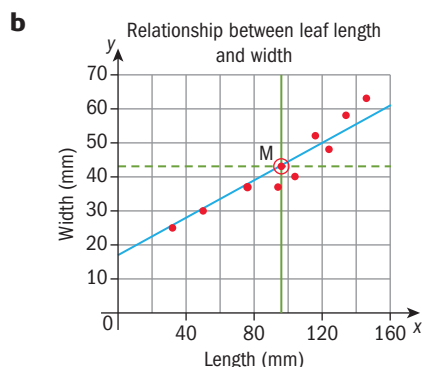
- b Strong, positive, linear.



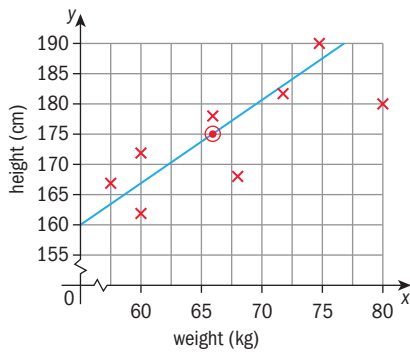
- a Strong, positive.  
 b The lean is increasing as the years increase.

#### Exercise 10B

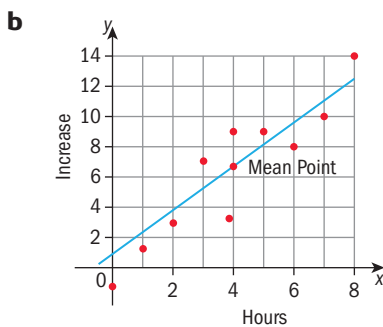
- 1 a Mean point = (mean of  $x$ , mean of  $y$ )  
 = (96.7, 44.1)



- 2 a i  $\frac{182 + 173 + 162 + 178 + 190 + 161 + 180 + 172 + 167 + 185}{10}$   
 $= 175 \text{ cm}$
- ii  $\frac{73 + 68 + 60 + 66 + 75 + 50 + 80 + 60 + 56 + 72}{10} = 66 \text{ kg}$



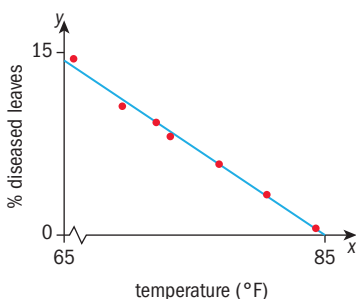
- 3 a Mean point = (mean of  $x$ , mean of  $y$ )  
 $= (4, 6.67)$



- c Strong, positive
- d An increase in the number of hours spent studying mathematics produces an increase in the grade.

### Exercise 10C

Temperature, ( $x$ ) °F	70	72	74	76	78	80
Percentage of diseased leaves, ( $y$ )	12.3	9.5	7.7	6.1	4.3	2.3



- a  $(\bar{x}, \bar{y}) = (75, 7.03)$
- b  $y = -0.96x + 79$
- c % diseased =  $(-0.96 \times \text{Temperature}) + 79$   
 % diseased =  $(-0.96 \times 75) + 79 = 7\%$

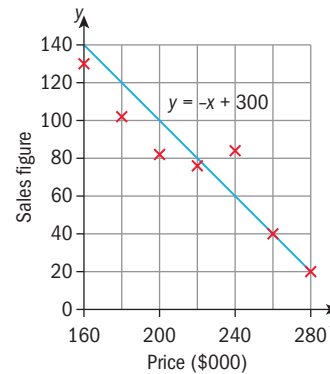
- 2 a Mean house price =  $\frac{\text{sum of prices}}{\text{number of prices}} = \frac{1540}{7} = 220$

The mean house price is \$220 000

- b Mean sales =  $\frac{\text{sum of sales}}{\text{number of sales}} = \frac{528}{7} = 75.4$

The mean number of sales is estimated at 75.4

- c and d Note the values of  $m$  and  $b$  in the equation  $y = mx + b$  are approximate.



- e Approximately 70 houses.

### Exercise 10D

- 1 The slope is  $-0.3$ . As a student plays one more day of sport per week they do 18 mins less homework per week.

The  $y$ -intercept is 40, which means that the average student who does no sport does 40 hours of homework per week.

- 2 The slope is 6. For every criminal a person knows, they will generally have been convicted of 6 times.

The  $y$ -intercept is 0.5, which means that people who do not know any criminals will, on average, have been convicted 0.5 times.

- 3 The slope is 2.4. For every pack of cigarettes smoked per week a person will, on average, take 2.4 more sick days per year.

The  $y$ -intercept is 7, which means that the average person that does not smoke has 7 sick days per year.

- 4 The slope is 100. 100 more customers come to his shop every year.

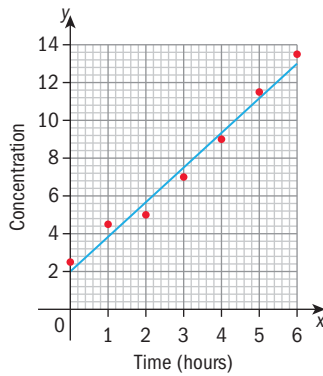
The  $y$ -intercept is  $-5$ , which means that  $-5$  people visited his shop in year zero, the  $y$ -intercept is not suitable for interpretation.

- 5 The slope is 0.8. Every 1 mark increase in mathematics results in a 0.8 increase in science.

The  $y$ -intercept is  $-10$  which is not suitable for interpretation as a zero in mathematics would mean a  $-10$  in science.

### Exercise 10E

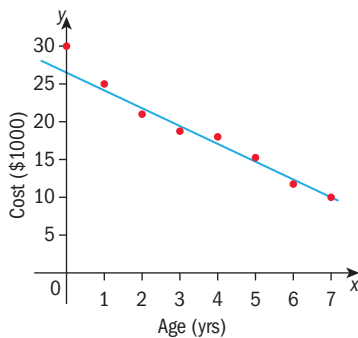
1 a



b  $y = 1.84x + 1.99$

c Concentration after 3.5 hours:  
 $y = 1.99 + 1.84 \times 3.5 = 8.43$

2 a

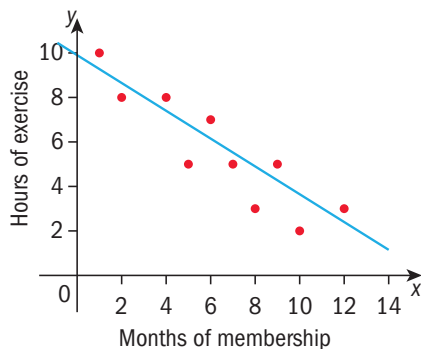


b  $y = -2.67x + 28.1$

c Cost =  $(-2.67 \times \text{Age}) + 28.1$   
 $= (-2.67 \times 4.5) + 28.1 = \$16\ 085$

d The relationship may not be linear. Old cars are often more expensive after 50 yrs than when new.

3 a



b  $y = -0.665x + 9.86$

c Hours of exercise =  $(-0.665 \times \text{months of membership}) + 9.86 = (-0.665 \times 3) + 9.86 = 7.865$  hrs.

d No. The equation gives  $-6.1$  hrs of exercise!

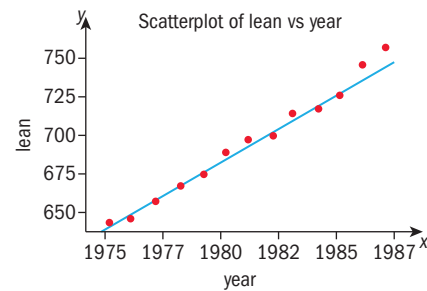
4 Fifty years = 600 months, and the line would predict Sarah's height at 50 years to be about 302 cm = 3.02 meters. Clearly there is a major difficulty with extrapolation. In fact, most females reach their maximum height in their mid to late teens, and from then on, their height is

fairly constant and therefore extrapolating with a linear function is unsuitable.

5 Revisit the data from the leaning tower of Pisa.

a (81, 694)

b



d  $y = 9.32x - 61.1$

e Lean =  $(9.32 \times \text{year}) - 61.1$   
 $= (9.32 \times 90) - 61.1 = 778$  cm.

### Exercise 10F

1  $r = 0.863$ . There is a strong, positive correlation.

2 a 0.789

b Strong, positive correlation.

c The income increases as the number of years of education increases.

3 a 0.910

b The stopping distance increases, as the car gets older.

c Strong positive correlation.

4 a  $-0.887$

b Strong, negative correlation.

c Yes, Kelly's grade would increase if the chat time decreased.

5 a 0.0262

b Positive, weak correlation.

c No. Mo's grade would not increase if the game time decreased.

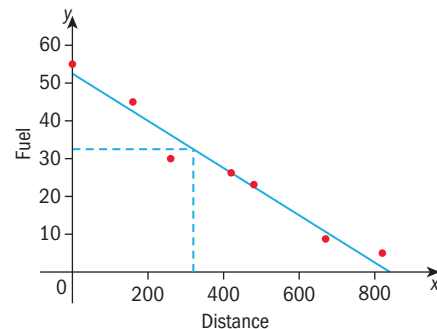
6  $r = 0.994$ . Strong, positive correlation.



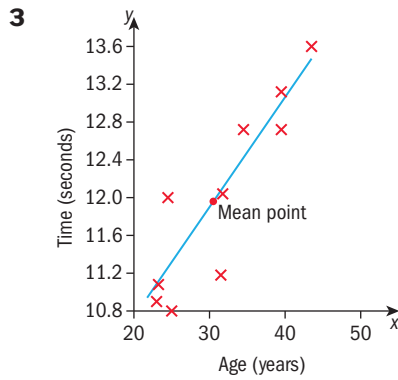
### Review exercise

1 a ii b v c iii d i

2 a



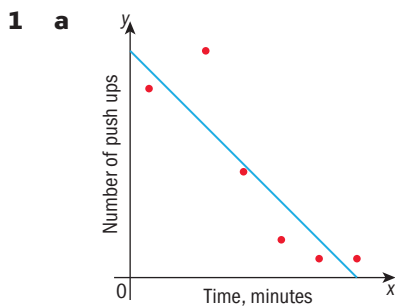
c 32. See the dotted lines on the graph.



- b** Mean Age =  $\frac{\text{sum of ages}}{\text{number of policemen}} = \frac{340}{10} = 34$   
 Mean time =  $\frac{\text{sum of times}}{\text{number of policemen}} = \frac{120}{10} = 12$   
 Mean age = 34 years. Mean time = 12 secs.  
**d** Approximately 11.9 secs. See the dotted lines on the scatterplot.

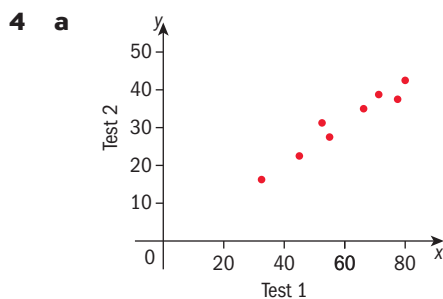


**Review exercise**

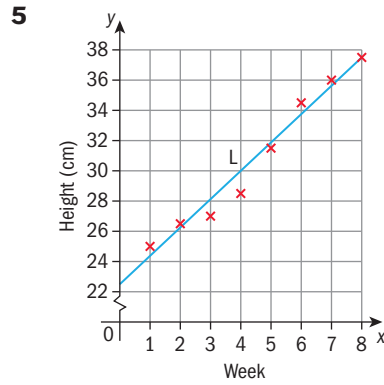


- b** As the time increase, the number of push-ups decreases.  
**c**  $y = -1.29x + 9$   
**d**  $r = -0.929$ . There is a strong, negative correlation.

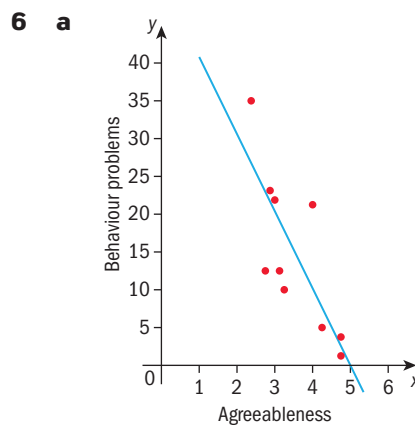
- 2 a**  $w = -22.4 + 55.5h$   
**b**  $w = -22.4 + 55.5 \times 1.6 = 66.4$  kg  
**3 a**  $r = 0.785$       **b**  $y = 30.7 + 0.688x$   
**c**  $IQ = 30.7 + (0.688 \times 100) = 99.5$ . This should be reasonably accurate since the product moment correlation coefficient shows fairly strong correlation.



- b** Positive, strong.  
**c** Students with a high score on test 1 tend to have a high score on test 2.  
**d**  $y = 0.50x + 0.48$   
**e** Test 2 Score =  $(0.50 \times \text{Test 1 score}) + 0.48$   
 $= (0.50 \times 40) + 0.48 = 20.48$



- b** (4, 30)  
**d i**  $r = 0.986$   
**ii** (very) strong positive correlation  
**e**  $y = 1.83x + 22.7$   
**g** Height =  $(1.83 \times \text{week number}) + 22.7$   
 $= (1.83 \times 4.5) + 22.7 = 30.9$  cm  
**g** Not possible to find an answer as the value lies too far outside the given set of data.



- b** Behavior problems decrease  
**c**  $-0.797$   
**d** Strong, negative correlation.  
**e** Teenagers who were more agreeable tended to have fewer behavior problems.  
**f**  $y = -10.2x + 51.0$   
**g** Number of behavior problems =  $(-10.2 \times \text{Agreeableness score}) + 51.0$   
 $= (-10.2 \times 4.5) + 51.0 = 3.1$

- 7 a**  $y = 10.7x + 121$  (3sf)  
**b i** Every coat on average costs \$10.65 to produce,  
**ii** When the factory does not produce any clothes then  $x = 0$ , it has to pay costs of \$121.  
**c** Cost =  $(10.7 \times 70) + 121 = \$866.67$   
**d**  $19.99x > 10.65x + 121$   
 $9.34x > 121$   
 $x > 12.995\dots$   
 13 coats should be produced in one day in order to make a profit.