Syllabus content

Topic 1—Algebra

9 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions. Examples include compound interest and population growth.	**Int:** The chess legend (Sissa ibn Dahir). **Int:** Aryabhatta is sometimes considered the “father of algebra”. Compare with al-Khawarizmi. **TOK:** How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof. **TOK:** Debate over the validity of the notion of “infinity”: finitists such as L. Kronecker consider that “a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps”. **TOK:** What is Zeno’s dichotomy paradox? How far can mathematical facts be from intuition?

	Content	Further guidance	Links
1.2	Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	*Examples*: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5\log 2$; $(2^3)^{-4} = 2^{-12}$. *Examples*: $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25}\left(= \frac{3}{2}\right)$. Link to 2.6, logarithmic functions.	**Appl:** Chemistry 18.1 (Calculation of pH). **TOK:** Are logarithms an invention or discovery? (This topic is an opportunity for teachers to generate reflection on "the nature of mathematics".)
1.3	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r}$. **Not required:** formal treatment of permutations and formula for ${}^nP_r$.	Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using **both** the formula and technology. *Example*: finding $\binom{6}{r}$ from inputting $y = 6^nC_rX$ and then reading coefficients from the table. Link to 5.8, binomial distribution.	**Aim 8:** Pascal's triangle. Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** The so-called "Pascal's triangle" was known in China much earlier than Pascal.

Topic 2—Functions and equations 24 hours

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$.

	Content	Further guidance	Links
2.1	Concept of function $f : x \mapsto f(x)$.		**Int:** The development of functions, Rene Descartes (France), Gottfried Wilhelm Leibniz (Germany) and Leonhard Euler (Switzerland).
	Domain, range; image (value).	*Example*: for $x \mapsto \sqrt{2-x}$, domain is $x \le 2$, range is $y \ge 0$. A graph is helpful in visualizing the range.	
	Composite functions.	$(f \circ g)(x) = f(g(x))$.	**TOK:** Is zero the same as "nothing"?
	Identity function. Inverse function f^{-1}.	$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.	**TOK:** Is mathematics a formal language?
	Not required: domain restriction.	On examination papers, students will only be asked to find the inverse of a *one-to-one* function.	
2.2	The graph of a function; its equation $y = f(x)$.		**Appl:** Chemistry 11.3.1 (sketching and interpreting graphs); geographic skills.
	Function graphing skills.	**Note** the difference in the command terms "draw" and "sketch".	**TOK:** How accurate is a visual representation of a mathematical concept? (Limits of graphs in delivering information about functions and phenomena in general, relevance of modes of representation.)
	Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.		
	Use of technology to graph a variety of functions, including ones not specifically mentioned.	An analytic approach is also expected for simple functions, including all those listed under topic 2.	
	The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.	Link to 6.3, local maximum and minimum points.	

	Content	Further guidance	Links
2.3	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations.	Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down. *Example*: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.	**Appl:** Economics 1.1 (shifting of supply and demand curves).
2.4	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x - h)^2 + k$, vertex (h, k).	Candidates are expected to be able to change from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.	**Appl:** Chemistry 17.2 (equilibrium law). **Appl:** Physics 2.1 (kinematics). **Appl:** Physics 4.2 (simple harmonic motion). **Appl:** Physics 9.1 (HL only) (projectile motion).

	Content	Further guidance	Links
2.5	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.		
	The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.	*Examples*: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$.	
	Vertical and horizontal asymptotes.	Diagrams should include all asymptotes and intercepts.	
2.6	Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$.		**Int:** The Babylonian method of multiplication: $ab = \frac{(a+b)^2 - a^2 - b^2}{2}$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.
	Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.	
	Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.		

	Content	Further guidance	Links
2.7	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2+bx+c=0$, $a\neq 0$. The quadratic formula. The discriminant $\Delta = b^2-4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solving exponential equations.	Solutions may be referred to as roots of equations or zeros of functions. Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions. *Examples*: $e^x = \sin x$, $x^4+5x-6=0$. *Example*: Find k given that the equation $3kx^2+2x+k=0$ has two equal real roots. *Examples*: $2^{x-1}=10$, $\left(\frac{1}{3}\right)^x = 9^{x+1}$. Link to 1.2, exponents and logarithms.	
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.	**Appl:** Compound interest, growth and decay; projectile motion; braking distance; electrical circuits. **Appl:** Physics 7.2.7–7.2.9, 13.2.5, 13.2.6, 13.2.8 (radioactive decay and half-life)

Topic 3—Circular functions and trigonometry 16 hours

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.	**Int:** Seki Takakazu calculating π to ten decimal places. **Int:** Hipparchus, Menelaus and Ptolemy. **Int:** Why are there 360 degrees in a complete turn? Links to Babylonian mathematics. **TOK:** Which is a better measure of angle: radian or degree? What are the "best" criteria by which to decide? **TOK:** Euclid's axioms as the building blocks of Euclidean geometry. Link to non-Euclidean geometry.
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is $y = x\tan\theta$. *Examples*: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan 210° = \frac{\sqrt{3}}{3}$.	**Aim 8:** Who really invented "Pythagoras' theorem"? **Int:** The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca. 510). **TOK:** Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?

	Content	Further guidance	Links
3.3	The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine. Relationship between trigonometric ratios.	Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities). *Examples*: Given $\sin\theta$, finding possible values of $\tan\theta$ without finding θ. Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.	
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs. Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Transformations. Applications.	*Examples*: $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos(3(x-4)) + 1$. *Example*: $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction. Link to 2.3, transformation of graphs. Examples include height of tide, motion of a Ferris wheel.	**Appl:** Physics 4.2 (simple harmonic motion).

	Content	Further guidance	Links
3.5	Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. **Not required:** the general solution of trigonometric equations.	*Examples*: $2\sin x = 1$, $0 \le x \le 2\pi$, $2\sin 2x = 3\cos x$, $0^{\circ} \le x \le 180^{\circ}$, $2\tan\left(3(x-4)\right) = 1$, $-\pi \le x \le 3\pi$. *Examples*: $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.	
3.6	Solution of triangles. The cosine rule. The sine rule, including the ambiguous case. Area of a triangle, $\frac{1}{2}ab\sin C$. Applications.	Pythagoras' theorem is a special case of the cosine rule. Link with 4.2, scalar product, noting that: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b} \Rightarrow \lvert \boldsymbol{c} \rvert^2 = \lvert \boldsymbol{a} \rvert^2 + \lvert \boldsymbol{b} \rvert^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$. Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**Aim 8:** Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** Cosine rule: Al-Kashi and Pythagoras. **TOK:** Non-Euclidean geometry: angle sum on a globe greater than 180°.

Topic 4—Vectors

16 hours

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

	Content	Further guidance	Links
4.1	Vectors as displacements in the plane and in three dimensions.	Link to three-dimensional geometry, x, y and z-axes.	**Appl:** Physics 1.3.2 (vector sums and differences) Physics 2.2.2, 2.2.3 (vector resultants). **TOK:** How do we relate a theory to the author? Who developed vector analysis: JW Gibbs or O Heaviside?
	Components of a vector; column representation; $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following:	Applications to simple geometric figures are essential.	
	• the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.	
	• multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;	Multiplication by a scalar can be illustrated by enlargement.	
	• magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	• unit vectors; base vectors; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$;		
	• position vectors $\vec{\mathrm{OA}} = \boldsymbol{a}$;		
	• $\vec{\mathrm{AB}} = \vec{\mathrm{OB}} - \vec{\mathrm{OA}} = \boldsymbol{b} - \boldsymbol{a}$.	Distance between points A and B is the magnitude of $\vec{\mathrm{AB}}$.	

	Content	Further guidance	Links
4.2	The scalar product of two vectors.	The scalar product is also known as the "dot product". Link to 3.6, cosine rule.	
	Perpendicular vectors; parallel vectors.	For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\boldsymbol{w} = k\boldsymbol{v}$, $\lvert \boldsymbol{v} \cdot \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert$.	
	The angle between two vectors.		
4.3	Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.	Relevance of $\boldsymbol{a}$ (position) and $\boldsymbol{b}$ (direction). Interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert \boldsymbol{b} \rvert$ representing speed.	**Aim 8:** Vector theory is used for tracking displacement of objects, including for peaceful and harmful purposes. **TOK:** Are algebra and geometry two separate domains of knowledge? (Vector algebra is a good opportunity to discuss how geometrical properties are described and generalized by algebraic methods.)
	The angle between two lines.		
4.4	Distinguishing between coincident and parallel lines. Finding the point of intersection of two lines. Determining whether two lines intersect.		

Topic 5—Statistics and probability 35 hours

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms “write down”, “find” and “calculate” will be used.

	Content	Further guidance	Links
5.1	Concepts of population, sample, random sample, discrete and continuous data.	Continuous and discrete data.	**Appl:** Psychology: descriptive statistics, random sample (various places in the guide).
	Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals;		**Aim 8:** Misleading statistics.
			Int: The St Petersburg paradox, Chebychev, Pavlovsky.
	box-and-whisker plots; outliers.	Outlier is defined as more than $1.5 \times \text{IQR}$ from the nearest quartile.	
		Technology may be used to produce histograms and box-and-whisker plots.	
	Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class.		
	Not required: frequency density histograms.		

	Content	Further guidance	Links
5.2	Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Applications.	On examination papers, data will be treated as the population. Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data. Calculation of standard deviation/variance using only technology. Link to 2.3, transformations. *Examples*: If 5 is subtracted from all the data items, then the mean is decreased by 5, but the standard deviation is unchanged. If all the data items are doubled, the median is doubled, but the variance is increased by a factor of 4.	**Appl:** Psychology: descriptive statistics (various places in the guide). **Appl:** Statistical calculations to show patterns and changes; geographic skills; statistical graphs. **Appl:** Biology 1.1.2 (calculating mean and standard deviation); Biology 1.1.4 (comparing means and spreads between two or more samples). **Int:** Discussion of the different formulae for variance. **TOK:** Do different measures of central tendency express different properties of the data? Are these measures invented or discovered? Could mathematics make alternative, equally true, formulae? What does this tell us about mathematical truths? **TOK:** How easy is it to lie with statistics?
5.3	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.	Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph.	

	Content	Further guidance	Links
5.4	Linear correlation of bivariate data. Pearson's product–moment correlation coefficient r. Scatter diagrams; lines of best fit. Equation of the regression line of y on x. Use of the equation for prediction purposes. Mathematical and contextual interpretation. **Not required:** the coefficient of determination R^2.	Independent variable x, dependent variable y. Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Positive, zero, negative; strong, weak, no correlation. The line of best fit passes through the mean point. Technology should be used find the equation. Interpolation, extrapolation.	**Appl:** Chemistry 11.3.3 (curves of best fit). **Appl:** Geography (geographic skills). Measures of correlation; geographic skills. **Appl:** Biology 1.1.6 (correlation does not imply causation). **TOK:** Can we predict the value of x from y, using this equation? **TOK:** Can all data be modelled by a (known) mathematical function? Consider the reliability and validity of mathematical models in describing real-life phenomena.
5.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $\mathrm{P}(A)=\frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes.	The sample space can be represented diagrammatically in many ways. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability. Simulations may be used to enhance this topic. Links to 5.1, frequency; 5.3, cumulative frequency.	**TOK:** To what extent does mathematics offer models of real life? Is there always a function to model data behaviour?

	Content	Further guidance	Links
5.6	Combined events, $\text{P}(A \cup B)$. Mutually exclusive events: $\text{P}(A \cap B) = 0$. Conditional probability; the definition $\text{P}(A \mid B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$. Independent events; the definition $\text{P}(A \mid B) = \text{P}(A) = \text{P}(A \mid B')$. Probabilities with and without replacement.	The non-exclusivity of “or”. Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae.	**Aim 8:** The gambling issue: use of probability in casinos. Could or should mathematics help increase incomes in gambling? **TOK:** Is mathematics useful to measure risks? **TOK:** Can gambling be considered as an application of mathematics? (This is a good opportunity to generate a debate on the nature, role and ethics of mathematics regarding its applications.)
5.7	Concept of discrete random variables and their probability distributions. Expected value (mean), $\text{E}(X)$ for discrete data. Applications.	Simple examples only, such as: $\text{P}(X = x) = \frac{1}{18}(4 + x)$ for $x \in \{1, 2, 3\}$; $\text{P}(X = x) = \frac{5}{18}, \frac{6}{18}, \frac{7}{18}$. $\text{E}(X) = 0$ indicates a fair game where X represents the gain of one of the players. Examples include games of chance.	

	Content	Further guidance	Links
5.8	Binomial distribution. Mean and variance of the binomial distribution. **Not required:** formal proof of mean and variance.	Link to 1.3, binomial theorem. Conditions under which random variables have this distribution. Technology is usually the best way of calculating binomial probabilities.	
5.9	Normal distributions and curves. Standardization of normal variables (z-values, z-scores). Properties of the normal distribution.	Probabilities and values of the variable must be found using technology. Link to 2.3, transformations. The standardized value (z) gives the number of standard deviations from the mean.	**Appl:** Biology 1.1.3 (links to normal distribution). **Appl:** Psychology: descriptive statistics (various places in the guide).

Topic 6—Calculus

40 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications.

	Content	Further guidance	Links
6.1	Informal ideas of limit and convergence.	*Example*: 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$. Technology should be used to explore ideas of limits, numerically and graphically.	**Appl:** Economics 1.5 (marginal cost, marginal revenue, marginal profit). **Appl:** Chemistry 11.3.4 (interpreting the gradient of a curve). **Aim 8:** The debate over whether Newton or Leibnitz discovered certain calculus concepts. **TOK:** What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? **TOK:** Opportunities for discussing hypothesis formation and testing, and then the formal proof can be tackled by comparing certain cases, through an investigative approach.
	Limit notation.	*Example*: $\lim_{x\to\infty}\left(\frac{2x+3}{x-1}\right)$ Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes.	
	Definition of derivative from first principles as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Link to 1.3, binomial theorem. Use of both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative.	
	Derivative interpreted as gradient function and as rate of change.	Identifying intervals on which functions are increasing or decreasing.	
	Tangents and normals, and their equations. **Not required:** analytic methods of calculating limits.	Use of both analytic approaches and technology. Technology can be used to explore graphs and their derivatives.	

	Content	Further guidance	Links
6.2	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Differentiation of a sum and a real multiple of these functions.		
	The chain rule for composite functions.	Link to 2.1, composition of functions.	
	The product and quotient rules.	Technology may be used to investigate the chain rule.	
	The second derivative.	Use of both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$.	
	Extension to higher derivatives.	$\frac{d^ny}{dx^n}$ and $f^{(n)}(x)$.	

	Content	Further guidance	Links
6.3	Local maximum and minimum points. Testing for maximum or minimum.	Using change of sign of the first derivative and using sign of the second derivative. Use of the terms "concave-up" for $f''(x) > 0$, and "concave-down" for $f''(x) < 0$.	**Appl:** profit, area, volume.
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	
	Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Optimization.	Both "global" (for large $\|x\|$) and "local" behaviour. Technology can display the graph of a derivative without explicitly finding an expression for the derivative. Use of the first or second derivative test to justify maximum and/or minimum values.	
	Applications. **Not required:** points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.	Examples include profit, area, volume. Link to 2.2, graphing functions.	

	Content	Further guidance	Links
6.4	Indefinite integration as anti-differentiation.		
	Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.	$\int \frac{1}{x} dx = \ln x + C$, $x > 0$.	
	The composites of any of these with the linear function $ax + b$.	*Example*: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	
	Integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$.	*Examples*: $\int 2x(x^2+1)^4 dx$, $\int x \sin x^2 dx$, $\int \frac{\sin x}{\cos x} dx$.	
6.5	Anti-differentiation with a boundary condition to determine the constant term.	*Example*: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	**Int:** Successful calculation of the volume of the pyramidal frustum by ancient Egyptians (Egyptian Moscow papyrus). Use of infinitesimals by Greek geometers.
	Definite integrals, both analytically and using technology.	$\int_a^b g'(x)dx = g(b) - g(a)$. The value of some definite integrals can only be found using technology.	Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui
	Areas under curves (between the curve and the x-axis). Areas between curves. Volumes of revolution about the x-axis.	Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.	**Int:** Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.
6.6	Kinematic problems involving displacement s, velocity v and acceleration a.	$v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.	**Appl:** Physics 2.1 (kinematics).
	Total distance travelled.	Total distance travelled $= \int_{t_1}^{t_2} \lvert v \rvert \, dt$.	