

$$1 \quad \mathbf{a} \quad \overrightarrow{AB} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$$

$$\mathbf{b} \quad 2 - 2\lambda = 6 + a\mu \quad (1)$$

$$-1 + 4\lambda = -5 - 3\mu \quad (2)$$

$$-5 + \lambda = 1 + \mu \quad (3)$$

$$(2) + 3 \times (3) \Rightarrow -16 + 7\lambda = -2$$

$$\lambda = 2, \quad \mu = -4$$

$$\text{sub. (1)} \quad 2 - 2(2) = 6 + a(-4)$$

$$-2 = 6 - 4a$$

$$a = 2$$

point of intersection: $(-2, 7, -3)$

$$2 \quad \mathbf{a} \quad \overrightarrow{BA} = (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$$

$$= -6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$$

$$\overrightarrow{BC} = (6\mathbf{i} + 4\mathbf{j} + \mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}) \\ = 4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$$

$$|\overrightarrow{BA}| = \sqrt{36 + 9 + 36} = 9$$

$$|\overrightarrow{BC}| = \sqrt{16 + 1 + 64} = 9$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = -24 + 3 + 48 = 27$$

$$\cos(\angle ABC) = \frac{27}{9 \times 9} = \frac{1}{3}$$

$$\mathbf{b} \quad \overrightarrow{AC} = (6\mathbf{i} + 4\mathbf{j} + \mathbf{k}) - (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \\ = 10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2} \overrightarrow{AC}$$

$$= (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + \frac{1}{2}(10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

$$= \mathbf{i} + 3\mathbf{j}$$

$$\mathbf{c} \quad \overrightarrow{BM} = (\mathbf{i} + 3\mathbf{j}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}) \\ = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$$

$$\overrightarrow{BM} \cdot \overrightarrow{AC} = (-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) \cdot (10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) \\ = -10 - 4 + 14 = 0$$

$\therefore BM$ perpendicular to AC

$$\mathbf{d} \quad \angle ABC = \cos^{-1} \frac{1}{3} = 70.529$$

isosceles triangle

$$\therefore \angle ACB = \frac{1}{2}(180 - 70.529) = 54.7^\circ \text{ (1dp)}$$

$$3 \quad \mathbf{a} \quad \overrightarrow{AB} = \begin{pmatrix} 11 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\therefore \mathbf{r} = \begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\mathbf{b} \quad \overrightarrow{OC} = \begin{pmatrix} 9+2\lambda \\ 5+2\lambda \\ -3 \end{pmatrix}$$

$$\overrightarrow{OC} \cdot \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 0$$

$$2(9+2\lambda) + 2(5+2\lambda) + 0 = 0$$

$$8\lambda + 28 = 0$$

$$\lambda = -\frac{7}{2}, \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$$

$$\mathbf{c} \quad OC = \sqrt{4+4+9} = \sqrt{17}$$

$$AC = \frac{7}{2} \left| \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \right| = 7\sqrt{1+1} = 7\sqrt{2}$$

$$\text{area} = \frac{1}{2} \times \sqrt{17} \times 7\sqrt{2} = 20.4$$

$$\mathbf{d} \quad AC = \frac{7}{2} AB$$

$$\therefore \text{area } OAB : \text{area } OAC = 2 : 7$$

$$4 \quad \mathbf{a} \quad |(7\mathbf{i} - 5\mathbf{j} - \mathbf{k})| = \sqrt{49+25+1} = 5\sqrt{3}$$

$$|(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})| = \sqrt{16+25+9} = 5\sqrt{2}$$

$$(7\mathbf{i} - 5\mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 28 + 25 - 3 = 50$$

$$\cos(\angle AOB) = \frac{50}{5\sqrt{3} \times 5\sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$$

$$\mathbf{b} \quad \overrightarrow{AB} = (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) - (7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$$

$$= -3\mathbf{i} + 4\mathbf{k}$$

$$\overrightarrow{AB} \cdot \overrightarrow{OB} = (-3\mathbf{i} + 4\mathbf{k}) \cdot (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$$

$$= -12 + 0 + 12 = 0$$

$\therefore AB$ perpendicular to OB

$$\mathbf{c} \quad \overrightarrow{OC} = \frac{3}{2}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$$

$$= 6\mathbf{i} - \frac{15}{2}\mathbf{j} + \frac{9}{2}\mathbf{k}$$

$$\overrightarrow{AC} = (6\mathbf{i} - \frac{15}{2}\mathbf{j} + \frac{9}{2}\mathbf{k}) - (7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$$

$$= -\mathbf{i} - \frac{5}{2}\mathbf{j} + \frac{11}{2}\mathbf{k}$$

$$\overrightarrow{AC} \cdot \overrightarrow{OA} = (-\mathbf{i} - \frac{5}{2}\mathbf{j} + \frac{11}{2}\mathbf{k}) \cdot (7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$$

$$= -7 + \frac{25}{2} - \frac{11}{2} = 0$$

$\therefore AC$ perpendicular to OA

$$\mathbf{d} \quad \angle CAO = 90^\circ$$

$$\therefore \angle ACO = 90^\circ - \angle AOC$$

$$= 90^\circ - \angle AOB$$

$$= 90^\circ - \cos^{-1}\left(\frac{1}{3}\sqrt{6}\right)$$

$$= 54.7^\circ$$