1 The points A and B have position vectors $\begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}$ respectively, relative to a fixed origin.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line m has equation

$$\mathbf{r} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} a \\ -3 \\ 1 \end{pmatrix},$$

where a is a constant.

Given that lines l and m intersect,

b find the value of a and the coordinates of the point where l and m intersect. **(6)**

2 Relative to a fixed origin, the points A, B and C have position vectors $(-4\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $(2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$ and $(6\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ respectively.

a Show that $\cos(\angle ABC) = \frac{1}{3}$. **(3)**

The point M is the mid-point of AC.

b Find the position vector of M. **(2)**

c Show that BM is perpendicular to AC. **(3)**

d Find the size of angle ACB in degrees. **(3)**

3 Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ 7 \\ -3 \end{pmatrix}$ respectively.

a Find, in vector form, an equation of the line L which passes through A and B. **(2)**

The point C lies on L such that OC is perpendicular to L.

b Find the position vector of C. **(5)**

c Find, to 3 significant figures, the area of triangle OAC. **(3)**

d Find the exact ratio of the area of triangle OAB to the area of triangle OAC. **(2)**

4 Relative to a fixed origin O, the points A and B have position vectors $(7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$ and $(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ respectively.

a Find $\cos(\angle AOB)$, giving your answer in the form $k\sqrt{6}$, where k is an exact fraction. **(4)**

b Show that AB is perpendicular to OB. **(3)**

The point C is such that $\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$.

c Show that AC is perpendicular to OA. **(3)**

d Find the size of $\angle ACO$ in degrees to 1 decimal place. **(3)**