# Mathematical Exploration – Math SL

## Exploration Time Frame:

It is envisaged that 10 hours of class time and approximately 10 hours outside class be spent on the exploration. The following is an outline of the stages of the Exploration.

*Choosing a focus/topic:*

* This will involve introductory lesson(s) leading to each student having a focused aim to their exploration.
* The purpose and scope of the exploration should be explained.
* A list of stimuli that students are encouraged to work from is provided, as well as an example of a “mind map” starting from the stimulus “water” is worked on to exemplify how this process could develop.

By the end of this stage, each student should have decided on a focus for the exploration and have a preliminary plan of how to approach it.

*Draft exploration:*

Discussion among their peers and with teacher is encouraged, but it is essential that the written draft exploration submitted is the student’s own work and they should be prepared to explain their work.

*Teacher to review and comment on drafts:*

* This draft should be reviewed by the teacher and comments made on the strengths and weaknesses of the work.
* This first draft will not be heavily annotated or edited by the teacher, but is an opportunity for students to receive further guidance on the exploration.

*Final writing:* Students finalize the exploration based on the draft and the advice given.

*Final marking:* Teacher will read and mark the exploration submitted by the student.

## Exploration Grading:

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics SL. A student’s mark on the exploration will count for 20% of the final IB mathematics grade. The other 80% will come from Paper I and Paper II.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

# Authenticity

# Be aware that the written work you submit must be entirely your own. When completing a piece of work outside the classroom, work independently. You are encouraged to seek help by referring to examples of your own work or to any available textbooks and/or by discussing particular issues with the teacher or with other students. However, if a classmate asks for help from you, you should not provide a direct answer but you should guide the classmate into productive routes of inquiry.

# You may wish to consult experts outside the school. However, it is not acceptable for you to receive written information on specific areas of a task or to have any part of the work completed for you. Generally speaking, talking through a task and discussing related mathematical concepts are acceptable.

# General comments:

# 1. On the cover page, type the following and sign below it if it is true:

# *Candidate declaration: I confirm that this work is my own work and is the final version. I have acknowledged each use of the words or ideas of another person, whether written, oral or visual.*

# *Candidate’s signature: ................................................................. Date: ..............................*

# 2. If you use any external sources or ideas that were not your own, provide complete references for these sources in the bibliography (MLA or APA format).

# 3. If you use computer software and/or a GDC to generate graphs or tables, type the following on the cover page:

# *In this assignment, I used (name of graphing software/model of GDC) to...*

# 4. You must be able to provide evidence that your submission is your own work and show that you worked on the task independently. Here are some suggestions on how to do this:

# a)  Bring the task to class whenever you are meant to be working on it, or make sure that your teacher sees your progress on the task periodically.

# b)  Save all rough work and staple it to the back of your final report. Be sure to clearly label it “Rough Work.”

# c)  Be ready to discuss the process and your results verbally or in writing if your teacher requires this after you submit your assignment.

#

# Outline of a Mathematical Investigation

*Aim: to develop a general statement based on a trend seen in data that you generate.*

1. **Introduction**
	1. Tell in general what the paper will be about.
	2. Define the problem and your variables.
2. **Observations (Data Collection)**
	1. Gather data using a clear mathematical strategy. You may have to do calculations to get your data. Your systematic choice of values to calculate will demonstrate your strategy.
	2. Organize your data – usually in a table or some sort of list or other organized way.
	3. Process and display the data, preferably using more than one form of mathematical representation (graph, diagram, equation). Label this diagram or graph in detail!
3. **Analysis**
	1. **Tell** the reader your steps/thinking. Describe with appropriate terminology and notation.
	2. **Show** the reader your steps/thinking. Use graphs, tables, diagrams and other visuals.
4. **Interpretation (General Statement)**

Write your general statement using correct mathematical terminology and notation.

1. **Validity (Testing the general statement)**
	1. Test the general statement in a variety of ways. For instance, if your general statement is an equation, you should substitute different **kinds** of numbers (0, positive numbers, negative numbers, decimals or fractions >1 or <1, irrational numbers like **π**) into the independent variable.
	2. Adjust your general statement if necessary and describe what you are changing and why.
	3. If you do not change your general statement, state why it does not need to be changed.
2. **Evidence (Justification/Proof)**
	1. Provide evidence throughout the paper, not just in one place. All examples should be placed with your text as you describe things to make it easier for the reader to understand.
	2. You can justify things in many ways – using examples or non-examples, by showing diagrams, or demonstrating patterns in tables of values.
3. **Scope and Limitations**
	1. If you found that some values don’t work (limitations) for your general statement, detail these. Perhaps only whole number values make sense, or negative numbers do not.
	2. **Scope** is written with **limitations** in mind. For example: $\{n:n\in Z, 0<n<30\}$
4. **Reflection and Conclusion**

Write about what you learned, other possible **related** problems that could be explored, or reflect on the correctness of your findings or the accuracy and precision used, or the impact of technology in your task. Suggest realistic improvements. Then “wrap it up.”

1. **Mathematical Terminology and Notation and Mathematical Communication**
	1. Check for appropriate math terminology (words).
	2. Check for appropriate math notation (symbols).
	3. Check for logical organization (introduction, conclusion). Check for coherence (logical development, easy to follow). Diagrams should be part of the paper, not at the end.

# Outline of Mathematical Modeling

*Aim: to develop a mathematical model of the data so we can use this model to make predictions.*

1. **Introduction**

Tell in general what the paper will be about. Define the problem and your variables.

1. **Observations (Data Collection and Preparation for Analysis)**
	1. Gather data using a clear mathematical strategy. You may have to do calculations to get your data. Your systematic choice of values to calculate will demonstrate your strategy.
	2. Organize your data – usually in a table or some sort of list.
	3. Process and display the data, preferably using more than one form of mathematical representation (graph, diagram, equation). Label this diagram or graph in detail, and define the variables and parameters in your equation.
2. **Analysis**
	1. Identify **trends** in the data (increasing or decreasing, how does it increase or decrease, end behavior, intercepts, asymptotes, maximum or minimum values, etc.)
	2. Discuss possible function model types (quadratic, exponential, logarithmic, trigonometric, rational, piecewise, etc.) that might fit the data.
	3. Choose a function model type and state why this fits best.
	4. **Tell** the reader your steps/thinking. Describe with appropriate terminology and notation.
	5. **Show** the reader your steps/thinking. Use graphs, tables, diagrams and other visuals.
3. **Interpretation (Equation of the Function Model)**
	1. Create a model analytically using your knowledge of parameters in the function type, using transformations, or using technology. Give the **domain** and **range** of this function.
	2. Write the equation of the function model using correct math terminology/notation.
4. **Goodness of Fit (Testing the Function Model)**
	1. Show your function model on a graph *with* the data. Compare the model to the data, and comment on how well the model fits the data. **Comment in context** wherever possible.
	2. Compare the model to another model found with regression or another technological tool.
5. **Applications, Modifications and Limitations**
	1. Modify your function model if necessary and describe what you are changing and why.
	2. If you do not modify your function model, state why it does not need to be changed.
	3. Apply the model to a different, yet related situation. **Comment in context** on how well the function models this situation. Explain modifications that would need to be made.
	4. **Scope** is written with **limitations** in mind. For example: $\{n:n\in Z, 0<n<30\}$
6. **Evidence (Justification/Proof)**
	1. Provide evidence throughout the paper, not just in one place. All examples should be placed with your text as you describe things to make it easier for the reader to understand.
	2. You can justify things in many ways – using examples or non-examples, by showing diagrams, by doing new calculations, or demonstrating patterns in tables of values.
7. **Reflection and Conclusion**

Write about what you learned, other possible **related** problems that could be explored, or reflect on the correctness of your findings or the accuracy and precision used, or the impact of technology in your task. Suggest realistic improvements. Then “wrap it up.”

1. **Mathematical Terminology and Notation and Mathematical Communication**
	1. Check for appropriate math terminology (words) and math notation (symbols).
	2. Check for logical organization (introduction, conclusion). Check for coherence (logical development, easy to follow). Diagrams should be part of the paper, not at the end.

# Language Usage\*

You can use this language for any report when you are interpreting any information.

\*special thanks to Ms. Griffin at PIADS for sharing!

**The Language of Observation**

When we observe, we SEE. We do not analyze or interpret…we look at what is actually there. We make our observations in simple, short sentences beginning:

*I observe… I discern… I can locate…*

*I see… I detect… I can identify...*

*I notice… I recognize…*

**The Language of Analysis**

When we analyse, we attempt to make sense of our observations; we try to figure out what our observations are telling us. We go beyond our initial observations and infer meaning. We create our analysis using sentences beginning:

*This creates a mood of… This reveals… This means…*

*This corresponds to… This evokes… This illustrates…*

*This helps one realize that… This shows… In other words…*

*This is in contradiction to… This demonstrates… This stands for…*

*This creates a feeling of… This symbolizes… This relates to…*

**The Language of Interpretation**

When we interpret, we attempt to make sense of our analysis; we look closely at our various pieces of analysis and find relationships among them. We try to figure out what we can conclude about the image as a whole.

*I would argue that… Therefore, …*

*We can conclude that… Clearly, then,…*

*From this, we can understand that … This tells us that …*

*From the evidence provided, it is evident that … Finally, …*

**The Language of Evidence**

When we make a point, we need to provide evidence (proof) for our ideas. This might mean that we use observations that we make or patterns we see as evidence. This might mean that we use a theory or another bit of knowledge from elsewhere. But we always provide evidence. And we always tell the reader how that evidence proves our idea.

*For example… Such as…*

*For instance… To illustrate…*

*Including… We can see this type of thing when we…*

# Some Helpful Vocabulary for this task

This is a list of related words that could be helpful. You are not required to use all of these words in your final product. This is not a list of all possible words – there might be more that you could use!

|  |  |  |  |
| --- | --- | --- | --- |
| *Domain* | *Range* | *Parameter* | *Variable* |
| *Axes (Set of Axes)* | *Model* | *Function Type* | *Regression Model* |
| *Analytical Model* | *Trend* | *Behavior* | *Fit* |
| *Periodic Function* | *Quadratic Function* | *Exponential Function* | *Piecewise Function* |
| *Logarithmic Function* | *Transformation* | *Translation* | *Stretch* |
| *Compression* | *Slope* | *Rate of Change* | *Validity* |
| *General Statement* | *Data* | *Scope* | *Limitations* |
| *Terminology* | *Notation* | *Forms.of.Representation* |  |

# Transitional Phrases\*

Use the following words in your essay to connect sentences, paragraphs and ideas.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **To add information** | **To conclude or summarize** | **To contrast or show difference** | **To show** **similarities** | **To emphasize a point** |
| AlsoFor ExampleAlong withAdditionallyAs wellAnotherBesidesAgain Furthermore | In shortFinallyIn conclusionConsequentlyDue toAs a resultThereforeThusSo | AlthoughHoweverDespite ButEven thoughEven soYetOn the other handOn the contraryStill | In the same mannerIn the same wayAlsoLikewiseLikeBoth AsSimilarly | AgainIndeedTo repeatIn factTo emphasize For this reasonWith this in mindSignificantly |

\*special thanks to Ms. Matsui for sharing!

## Change Relationships

As \_\_\_\_\_\_\_\_\_ increases/decreases by \_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_ increases by \_\_/decreases by \_\_/stays the same.

Example: In the equation *f(x) = ax2 + bx + c,* as the absolute value of *a* increases, the slope of the parabola gets steeper.

## Direct or Indirect Relationships

If \_\_\_ is equal to \_\_\_\_, then \_\_\_ is equal to \_\_\_.

Example: In the equation *f(x) = ax2 + bx + c,* when I make the *c* value equal to 3, I notice that the *y*-intercept is 3.

Example: In the equation *f(x) = a(x – p)(x – q),* when *p* is -1 and *q* is 4, the *x*-intercepts of the parabola are -1 and 4.

# Submitting your Task

Please submit your task as a **Word Document.** Follow these instructions:

1. Make sure the file name is:

**Class Lastname Firstname Exploration.docx**

For Example:

**SL11 Kim Michael Exploration.docx**

1. Please e-mail to **michaelkim@branksome.asia**
2. Keeping a hard copy for your own records is a good idea.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion A: Communication | Criterion B: Mathematical Presentation | Criterion C: Personal Engagement | Criterion D: Reflection | Criterion E: Use of Mathematics |
|  | **Authentic Exploration Criteria** |
| 0 | T h e e x p l o r a t i o n d o e s n o t r e a c h t h e s t a n d a r d d e s c r i b e d b y t h e d e s c r i p t o r s b e l o w |
| 1 | The exploration has **some** **coherence** | There is **some** **appropriate** mathematical presentation | There is evidence of **limited or superficial** personal engagement | There is evidence of **limited or superficial** reflection | * **Some relevant** mathematics is used
 |
| 2 | The exploration has some coherence and **shows some organization** | The mathematical presentation is **mostly appropriate** | There is evidence of **some** personal engagement | There is evidence of **meaningful** reflection | * **Some relevant** mathematics is used
* **Limited understanding** is demonstrated
 |
| 3 | The exploration is **coherent and well-organized** | The mathematical presentation is **appropriate throughout** | There is evidence of **significant** personal engagement | There is **substantial evidence** of **critical** reflection | * Relevant mathematics commensurate with the level in the course is used.
* **Limited understanding** is demonstrated
 |
| 4 | The exploration is coherent, well-organized, **concise, and complete** |  | There is **abundant evidence** of **outstanding** personal engagement |  | * Relevant mathematics commensurate with the level in the course is used.
* The mathematics explored is **partially correct.**
* **Some** knowledge and understanding are demonstrated
 |
| 5 |  |  |  |  | * Relevant mathematics commensurate with the level in the course is used.
* The mathematics explored is **mostly correct**.
* **Good** knowledge and understanding are demonstrated.
 |
| 6 |  |  |  |  | * Relevant mathematics commensurate with the level in the course is used.
* The mathematics explored is **correct**.
* **Thorough** knowledge and understanding are demonstrated.
 |
| Comment |  |  |  |  |  |

**Score:** /7

**Total:** /20

# Notes on the Criteria

**Criterion A: Communication**

A well-organized exploration includes an introduction, has a rationale (which includes explaining why the topic was chosen), describes the aim of the task and has a conclusion. A coherent task is logically developed and easy to follow.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document.

This criterion includes, but is not limited to:

* Expressing ideas clearly
* Identifying a clear aim for the exploration
* Focusing on the aim and avoiding irrelevance
* Structuring ideas in a logical manner
* Editing the exploration so that it is easy to follow
* Citing references where appropriate
* Including an introduction, rationale, description of aims, conclusion
* Making sure that graphs, tables and diagrams accompany the work in the appropriate place and are not attached as appendices to the document.

**Criterion B: Mathematical Presentation**

This criterion assesses to what extent the student is able to:

* use appropriate mathematical language (notation, symbols, terminology),
* define key terms, where required
* use multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate.

Students are expected to use mathematical language when communicating mathematical ideas, reasoning and findings.

Students are encouraged to choose and use appropriate ICT tools such as graphic display calculators, screenshots, graphs, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical presentation and communication.

This criterion includes, but is not limited to:

* Using appropriate mathematical language and representation (notation, symbols, terminology).
* Defining key terms, where required.
* Selecting appropriate mathematical tools (including information and communication technology).
* Expressing results to an appropriate degree of accuracy.
* Using multiple forms of mathematical representation, such as formulae, diagrams,  tables, charts, graphs and models, where appropriate.
* Using mathematical language when communicating  mathematical ideas, reasoning and findings.
* Choosing and using appropriate ICT tools such as graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical communication.

**Criterion C: Personal Engagement**

This criterion assesses the extent to which the student engages with the exploration and makes it their own. Personal engagement may be recognized in different attributes and skills. These include thinking independently and/or creatively, addressing personal interest and presenting mathematical ideas in their own way.

This criterion includes, but is not limited to:

* Working independently.
* Asking questions, making conjectures and investigating mathematical ideas.
* Reading mathematics and researching areas of interest.
* Looking for and creating mathematical models for real-world situations.
* Considering historical and global perspectives.
* Thinking independently and/or creatively.
* Addressing personal interest.
* Presenting mathematical ideas in their own way.
* Evidence that the student makes the work their own, not just simply a written  report.
* The use of technology that significantly enhances the development of the task.
* Exploration of a further problem, or suggestion of further avenues for exploration connected with the context of this exploration.
* Meaningful and critical reflection on their work on the task and their approach to solving the problem
* Asking and answering questions: “I wonder if...”, “What would happen if...” “Why

does that happen?”

One main question the moderator will ask themselves when marking your work is, “Can I hear the student’s voice?” Keep this in mind.

**Criterion D: Reflection**

This criterion assesses how the student reviews, analyses, and evaluates the exploration. Although reflection may be seen in the conclusion of the exploration, it may also be found throughout the exploration.

This criterion includes but is not limited to:

* How the student reviews, analyses and evaluates the exploration.
* Discussing the implications of results
* Considering the significance of the exploration
* Looking at possible limitations and/or extensions
* Making links to different fields and/or areas of mathematics

**Criterion E: Use of Mathematics**

This criterion assesses to what extent students use mathematics in the exploration.

Students are expected **to produce work that is commensurate with the level in the course**. The mathematics explored should either be part of the syllabus, or at a similar level or beyond. It should not be completely based on mathematics listed in the prior learning. If the level of mathematics is not commensurate with the level in the course, **a maximum of two marks can be awarded for this criterion**.

The mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

This criterion includes, but is not limited to:

* Demonstrating knowledge and understanding
* Producing work that is within the level of the course. It should not be solely based  on mathematics listed in the prior learning.
* Applying mathematics in different contexts
* Applying problem-solving techniques
* Recognizing and explaining patterns, where appropriate
* This criterion assesses to what extent students use mathematics in the exploration.
* Sophistication in mathematics may include understanding and use of challenging  mathematical concepts, looking at a problem from different perspectives and seeing  underlying structures to link different areas of mathematics.
* Rigour involves clarity of logic and language when making mathematical arguments  and calculations.
* Precise mathematics is error-free and uses an appropriate level of accuracy at all  times.

***Note****: If the level of mathematics is not commensurate with the level of the course,* ***a maximum of two marks*** *can be awarded.*

***Note 2****: A piece of mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.*

***Note 3****: This is the only criterion that should differ from SL to HL.*