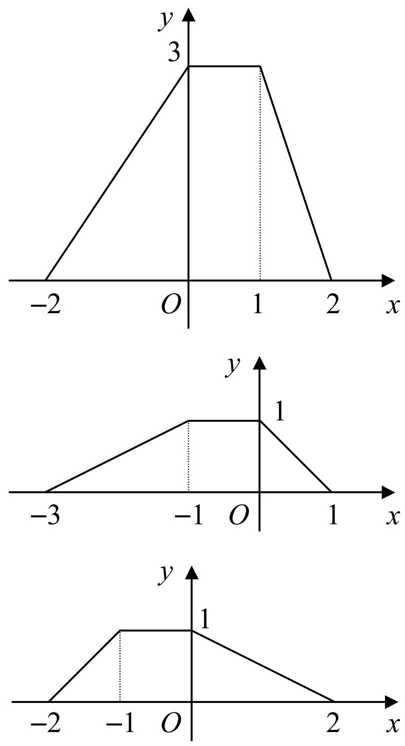
1 **a** $4x^2 - 9x + 5 = 3x - 4$
$4x^2 - 12x + 9 = 0$
$(2x - 3)^2 = 0$
$x = \frac{3}{2}$
$\therefore\ x = \frac{3}{2}, y = \frac{1}{2}$

b $y = 3x - 4$ is a tangent to the curve $y = 4x^2 - 9x + 5$ at the point $(\frac{3}{2}, \frac{1}{2})$

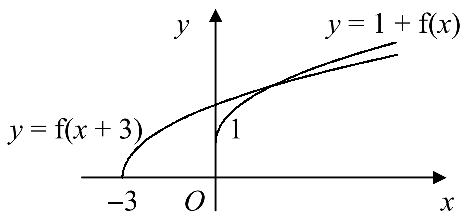
2 **a**



3 **a** $x^2 + 5x + 2 = 4x + 1$
$x^2 + x + 1 = 0$
$b^2 - 4ac = 1 - 4 = -3$
$b^2 - 4ac < 0\ \therefore$ no real roots
$\therefore$ does not intersect

b $x^2 + 5x + 2 = mx + 1$
$x^2 + (5 - m)x + 1 = 0$
only one root $\therefore\ b^2 - 4ac = 0$
$(5 - m)^2 - 4 = 0$
$5 - m = \pm 2$
$m = 3$ or 7

4 **a**

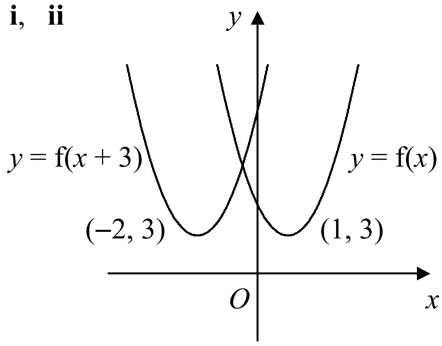


b $1 + \sqrt{x} = \sqrt{x+3}$
$(1 + \sqrt{x})^2 = x + 3$
$1 + 2\sqrt{x} + x = x + 3$
$\sqrt{x} = 1$
$x = 1\ \therefore\ (1, 2)$

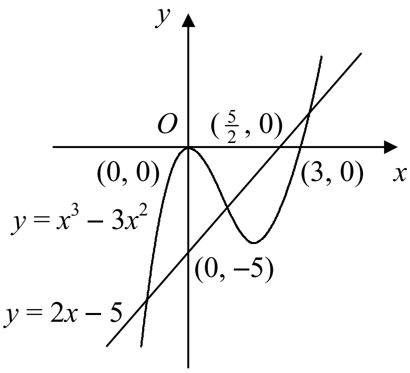
5 $x^2 + kx - 3 = k - x$
$x^2 + (k + 1)x - (k + 3) = 0$
$b^2 - 4ac = (k + 1)^2 + 4(k + 3)$
$= k^2 + 6k + 13$
$= (k + 3)^2 - 9 + 13$
$= (k + 3)^2 + 4$
real $k \Rightarrow (k + 3)^2 \geq 0$
$\Rightarrow (k + 3)^2 + 4 \geq 4$
$\therefore\ b^2 - 4ac > 0$
$\Rightarrow$ real and distinct roots
$\therefore$ l intersects C at exactly two points

6 **a** $f(x) = 2[x^2 - 2x] + 5$
$= 2[(x - 1)^2 - 1] + 5$
$= 2(x - 1)^2 + 3$

b **i**, **ii**



7 **a** $y = x^3 - 3x^2 = x^2(x - 3)$



b 3 real roots

$x^3 - 3x^2 - 2x + 5 = 0 \Rightarrow x^3 - 3x^2 = 2x - 5$

the graphs of $y = x^3 - 3x^2$ and $y = 2x - 5$

intersect at three points

8 touches x-axis at (2, 0)

$\therefore\ y = k(x - 2)^2$

crosses y-axis at (0, –6)

$\therefore\ -6 = 4k$

$k = -\frac{3}{2}$

$\therefore\ y = -\frac{3}{2}(x - 2)^2$

$y = -\frac{3}{2}x^2 + 6x - 6$

$\therefore\ a = -\frac{3}{2}$, $b = 6$ and $c = -6$

9 **a** LHS $= (1 - x)(2 + x)^2$

$= (1 - x)(4 + 4x + x^2)$

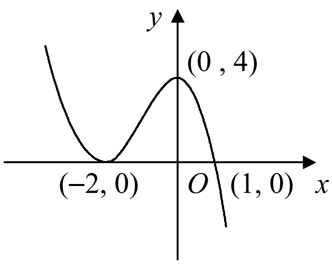
$= (4 + 4x + x^2) - x(4 + 4x + x^2)$

$= 4 + 4x + x^2 - 4x - 4x^2 - x^3$

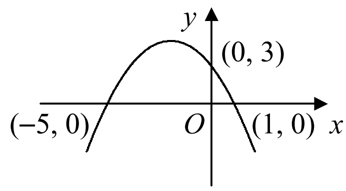
$= 4 - 3x^2 - x^3$

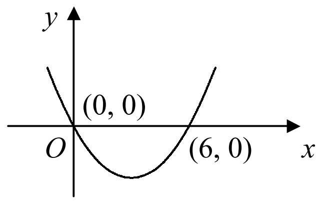
= RHS

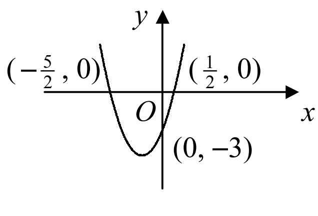
b



10 **a**

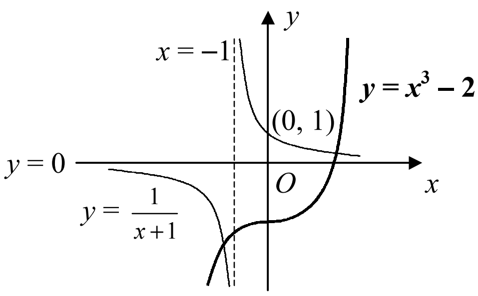






11 **a** translation by 1 unit in the negative x-direction

b



c $x^3 - \frac{1}{x+1} = 2 \Rightarrow x^3 - 2 = \frac{1}{x+1}$

the graphs $y = x^3 - 2$ and $y = \frac{1}{x+1}$ intersect

at one point for $x > 0$ and at one point for $x < 0$

$\therefore$ one positive and one negative real root