Functions

Domain is the set of all input values (x-values) for a function. Generally it is limited by the following illegal operations:

- -division by zero
- -square roots of negative numbers
- -logs of zero or negatives
- -certain values for tanΘ

Range is corresponding set of output values (y-values) for a function. The graph and/or knowledge of asymptotes, maxima, minima should help determine the range.

Consider the following functions:

$f(x) = 2x - 3$ $g(x) = 4^x$ $h(x) = x^2 - 1$

a) $g(\frac{5}{2})$

b) $(h \circ g)(-2)$

c) $f^{-1}(x)$

d) solve $h(x) = 0$

GDC skills

You must be able to reproduce a graph from your GDC and accurately show appropriate domain and range, maxima/minima, asymptotes, and axis-intercepts.

You need to be able to solve any equation graphically.

Families of functions

You should know the basic shapes and properties of the following graphs.

$y = x$ $\quad y = x^2$ $\quad y = \frac{1}{x}$

$y = e^x$ $\quad y = \ln x$

Transformations

For a function y = *f(x)*, the following rules define certain transformations:

$f(x-c)$ translate (shift) right *c* units

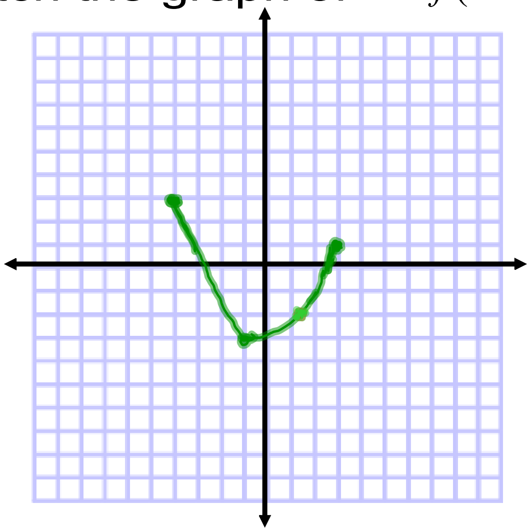
$f(x)+d$ translate (shift) up *d* units

$pf(x)$ stretch vertically by a scale factor of *p*
(note: negative *p* values cause a reflection through the x-axis)

$f(qx)$ stretch horizontally by a scale factor of 1/q
(note: negative *q* values cause a reflection through the y-axis)

**These may be combined

Ex.) Consider the graph of f(x) shown below. On the same grid, sketch the graph of $-2f(x+3)$.



The point A (3, 1) is on f(x). Write the coordinates of the image of A on $f(2x)-3$.

Ex.) The function g(x) = -f(x-3)+1. Give a full geometric description of the transformation from *f* to *g*.

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Ex. The function $h(x) = f\left(\frac{1}{3}x\right)$. Give a full geometric description of the transformation for *f* to *h*.

General Concepts

The y-intercept of a function can be found by subbing 0 for *x*.

The x-intercept(s) of a function (if any) can be found by subbing 0 for *y*.

Quadratic functions

Quadratic graphs are parabolae. There are three popular forms to write them:

1) "Vertex Form" $y = a(x-h)^2 + k$

-the vertex is at (*h, k*)
-the a-value determines the direction of opening and steepness

2) "Factored Form" $y = a(x-p)(x-q)$

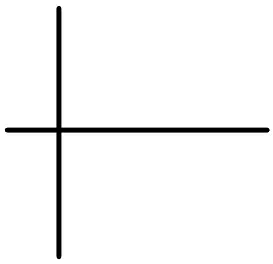
-the values of *p* and *q* are the x-intercepts
-the vertex and the axis of symmetry lie halfway between the x-intercepts
-the a-value determines the direction of opening and steepness
-parabolae that do not have x-intercepts cannot be written in this form

3) "Trinomial Form" $y = ax^2 + bx + c$

-*c* is the y-intercept
-the a-value determines the direction of opening and steepness
-the axis of symmetry is $x = \frac{-b}{2a}$ (in formula booklet)
-can be converted to "vertex form" through completing the square, using the axis of symmetry, or using differentiation techniques to find maxima/minima

Ex.) Convert $y = -2x^2 + 6x - 11$ to the form $y = -2(x - p)^2 + q$

Ex.) The graph alongside is a parabola with x-intercepts at -1 and 5 and has an equation y = a(x-m)(x-n).



a) Write down the values of *m* and *n*.

b) There is a minimum at (*p*, -36). Find the value of *p*.

c) Hence, find the equation of the parabola, giving your final answer in the form $y = ax^2 + bx + c$

Solving Quadratic Equations

For easily factorable quadratics, factorisation is often the best route.

$$2x^2 - 3x - 5 = 0$$

Alternately, the quadratic formula or a GDC may be used (GDC is preferable on paper 2)

$$\frac{x^2 + 1}{x + 1} = 3$$

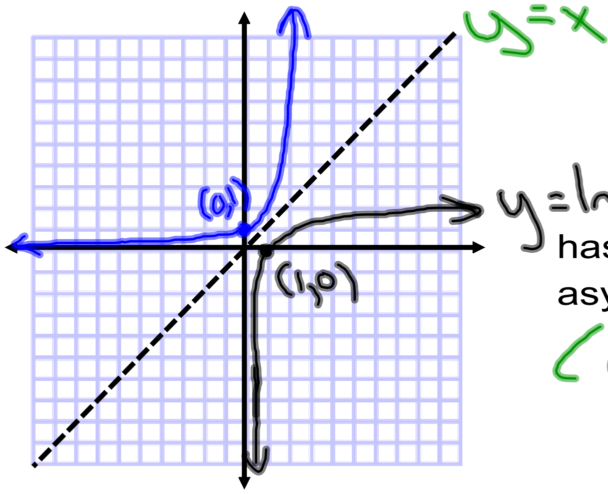
The Discriminant $\Delta = b^2 - 4ac$

In a quadratic equation, the discriminant tells us how many solutions exist (0, 1, or 2). In a quadratic function, the discriminant tells us how many x-intercepts exist (0, 1, or 2).

If $\Delta<0$, there are no x-intercepts or solutions.
If $\Delta=0$, there is one (repeated) x-intercept or solution.
If $\Delta>0$, there are 2 x-intercepts or solutions.

Ex.) The function $f(x) = x^2 - 5x + 2k$ has no x-intercepts. State the possible values of *k*.

By investigating the graphs of exponential and logarithmic functions, we can see that they are inverses of each other.



$y = e^x$ has a horizontal asymptote of y = 0 (x-axis)

$y = \ln x$ has a vertical asymptote of x = 0 (y-axis)

Ex.) If $f(x) = 2^{3x} - 5$, find $f^{-1}(x)$

Ex.) If $g(x) = \ln(2x - 5) + 1$, find $g^{-1}(x)$