## C3

1

$$
\begin{equation*}
\mathrm{f}: x \rightarrow 2+\log _{4} x, x \in \mathbb{R}, x>0 \tag{3}
\end{equation*}
$$

a Evaluate $\mathrm{ff}(1)$.
b Solve the equation $\mathrm{f}(x)=0$.
c Find the inverse function $\mathrm{f}^{-1}(x)$.
2 The function f is defined by

$$
\mathrm{f}: x \rightarrow|3 x-a|, x \in \mathbb{R}
$$

where $a$ is a positive constant.
a Find $\mathrm{ff}(-2 a)$.
b Sketch the graph $y=\mathrm{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes.
c Solve the equation $\mathrm{f}(x)=x$, giving your answers in terms of $a$.


The diagram shows the graph of $y=\mathrm{f}(x)$ which meets the $x$-axis at the point $\left(\frac{9}{4}, 0\right)$ and the $y$-axis at the point $(0,-3)$.
a Sketch on separate diagrams the graphs of
i $y=|\mathrm{f}(x)|$,
ii $y=\mathrm{f}^{-1}(x)$.
Given that $\mathrm{f}(x)$ is of the form $\mathrm{f}(x) \equiv a x^{\frac{1}{2}}+b, x \in \mathbb{R}, x \geq 0$,
b find the values of the constants $a$ and $b$,
c find an expression for $\mathrm{f}^{-1}(x)$.
4 The function f is defined by

$$
\begin{equation*}
\mathrm{f}: x \rightarrow \frac{x+2}{x-1}, x \in \mathbb{R}, x \neq 1 \tag{3}
\end{equation*}
$$

a Show that $\mathrm{ff}(x)=x$ for all $x \in \mathbb{R}, x \neq 1$.
b Hence, write down an expression for $\mathrm{f}^{-1}(x)$.
The function g is defined by

$$
\mathrm{g}: x \rightarrow 2 x-3, x \in \mathbb{R}
$$

c Solve the equation $\operatorname{gf}(x)=0$.
5 a Sketch on the same set of axes the graphs of $y=|x|$ and $y=|2 x-3|$.
b Hence, or otherwise, solve the equation

$$
\begin{equation*}
|x|=|2 x-3| \tag{4}
\end{equation*}
$$

6 The function $\mathrm{f}(x)$ is defined for all real values of $x$ by

$$
\begin{array}{ll}
\mathrm{f}(x)=x+2, & x<1 \\
\mathrm{f}(x)=4-x^{2}, & x \geq 1
\end{array}
$$

a Sketch the graph of $\mathrm{f}(x)$ showing the coordinates of any points of intersection with the coordinate axes.
b Evaluate $\mathrm{ff}(3)$.
c Solve the equation $\mathrm{f}(x)=1$.
7 The functions f and g are defined by

$$
\begin{aligned}
& \mathrm{f}: x \rightarrow k x+2, x \in \mathbb{R} \\
& \mathrm{~g}: x \rightarrow x-3 k, x \in \mathbb{R}
\end{aligned}
$$

where $k$ is a constant.
a Find expressions in terms of $k$ for
i $\mathrm{f}^{-1}(x)$,
ii $\operatorname{fg}(x)$.
Given that $\operatorname{fg}(7)=4$,
b find the two possible values of $k$.

$$
\begin{equation*}
\mathrm{f}(x) \equiv x^{2}-4 x+5, x \in \mathbb{R}, x \geq 2 \tag{2}
\end{equation*}
$$

a Express $\mathrm{f}(x)$ in the form $a(x+b)^{2}+c$.
b State the range of f .
c Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain.
d Sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ on the same diagram and state the relationship between the graphs.

9 The functions $f$ and $g$ are defined by

$$
\begin{aligned}
& \mathrm{f}: x \rightarrow x^{2}+4, x \in \mathbb{R} \\
& \mathrm{~g}: x \rightarrow 2 x-\frac{1}{x}, x \in \mathbb{R}, x \neq 0
\end{aligned}
$$

a Evaluate $\mathrm{gf}(-2)$.
b Find and simplify an expression for $\operatorname{fg}(x)$.
c Find the values of $x$ for which $\operatorname{fg}(x)=5$.
10 The function $f$ is given by

$$
\mathrm{f}: x \rightarrow \mathrm{e}^{\frac{1}{2} x}-3, x \in \mathbb{R}
$$

a Find $\mathrm{f}^{-1}(x)$ and state its domain.
b Sketch the curve $y=\mathrm{f}^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes.
The function g is given by

$$
\mathrm{g}: x \rightarrow \ln (x+5), x \in \mathbb{R}, x>-5
$$

c Evaluate $\mathrm{fg}(4)$.
d Solve the equation $\mathrm{f}^{-1}(x)=\mathrm{g}(x)$.

