**Exam Review Functions and Equations**

**1.** Part of the graph of a function *f* is shown in the diagram below.



(a) On the same diagram sketch the graph of *y* = − *f* (*x*).

(2)

(b) Let *g* (*x*) = *f* (*x* + 3).

(i) Find *g* (−3).

(ii) Describe **fully** the transformation that maps the graph of *f* to the graph of *g*.

 (4)

(Total 6 marks)

**2.** The following diagram shows part of the graph of *f*, where *f* (*x*) = *x*2 − *x* − 2.



(a) Find both *x*-intercepts.

(4)

(b) Find the *x*-coordinate of the vertex.

 (2)

(Total 6 marks)

**3.** Let *f* (*x*) = ln (*x* + 5) + ln 2, for *x*  –5.

(a) Find *f* −1(*x*).

(4)

Let *g* (*x*) = e*x*.

(b) Find (*g* ◦ *f*) (*x*), giving your answer in the form *ax* + *b*, where *a*, *b*, .

 (3)

(Total 7 marks)

**4.** (a) Consider the equation 4*x*2 + *kx* + 1 = 0. For what values of *k* does this equation have two **equal** roots?

(3)

Let *f* be the function *f* (** ) = 2 cos 2** + 4 cos ** + 3, for −360  **  360.

(b) Show that this function may be written as *f* (** ) = 4 cos2 ** + 4 cos ** + 1.

(1)

(c) Consider the equation *f* (** ) = 0, for −360  **  360.

(i) How many distinct values of cos ** satisfy this equation?

(ii) Find all values of ** which satisfy this equation.

(5)

(d) Given that *f* (** ) = *c* is satisfied by only three values of **, find the value of *c*.

(2)

(Total 11 marks)

**5.** The area *A* km2 affected by a forest fire at time *t* hours is given by *A* = *A*0 e*kt*.

When *t* = 5, the area affected is 1 km2 and the rate of change of the area is 0.2 km2 h−1.

(a) Show that *k* = 0.2.

(4)

(b) Given that *A*0 = , find the value of t when 100 km2 are affected.

 (2)

(Total 6 marks)

**6.** Let *f* (*x*) = log*a* *x*, *x*  0.

(a) Write down the value of

(i) *f* (*a*);

(ii) *f* (1);

(iii) *f* (*a*4 ).

 (3)

(b) The diagram below shows part of the graph of *f*.



On the same diagram, sketch the graph of *f*−1.

(3)

(Total 6 marks)

**7.** (a) Given that (2*x*)2 + (2*x*) −12 can be written as (2*x* + *a*)(2*x* + *b*), where *a*, *b*  , find the value of *a* and of *b*.

(b) Hence find the **exact** solution of the equation (2*x*)2 + (2*x*) −12 = 0, and explain why there is only one solution.

 (Total 6 marks)

**8.** The function *f* is defined as *f* (*x*) = (2*x* +1) e−*x*, 0  *x*  3. The point P(0, 1) lies on the graph of *f* (*x*), and there is a maximum point at Q.

(a) Sketch the graph of *y* = *f* (*x*), labelling the points P and Q.

(3)

(b) (i) Show that *f* ′ (*x*) = (1− 2*x*) e−*x*.

(ii) Find the **exact** coordinates of Q.

(7)

(c) The equation *f* (*x*) = *k*, where *k*  , has two solutions. Write down the range of values of *k*.

(2)

(d) Given that *f* (*x*) = e−*x* (−3 + 2*x*), show that the curve of *f* has only one point of inflexion.

(2)

(e) Let R be the point on the curve of *f* with *x*-coordinate 3. Find the area of the region enclosed by the curve and the line (PR).

(7)

(Total 21 marks)

**9.** (a) Express y = 2*x*2 – 12*x* + 23 in the form *y* = 2(*x* – *c*)2 + *d*.

The graph of *y* = *x*2 is transformed into the graph of *y* = 2*x*2 – 12*x* + 23 by the transformations

 a vertical stretch with scale factor *k* **followed by**

 a horizontal translation of *p* units **followed by**

 a vertical translation of *q* units.

(b) Write down the value of

(i) *k*;

(ii) *p*;

(iii) *q*.

 (Total 6 marks)

**10.** Solve the following equations.

(a) ln (*x* + 2) = 3.

(b) 102*x* = 500.

 (Total 6 marks)

**11.** Consider the function *f* (*x*) =  + 8, *x*  10.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) *y*-intercept;

(ii) *x*-intercept.

(2)

(c) Sketch the graph of *f* , clearly showing the above information.

(4)

(d) Let *g* (*x*) = , *x*  0.

The graph of *g* is transformed into the graph of *f* using two transformations.

 The first is a translation with vector  Give a full geometric description of the second transformation.

(2)

(Total 10 marks)