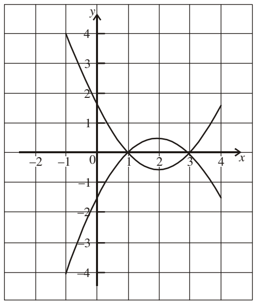
1. (a)



M1A1 N2

Note: *Award M1 for evidence of reflection in x-axis, A1 for correct vertex and all intercepts approximately correct.*

(b) (i) $g(-3) = f(0)$ (A1)
$f(0) = -1.5$ A1 N2

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ A1A1 N2

[6]

2. (a) evidence of attempting to solve $f(x) = 0$ (M1)
evidence of correct working A1

eg $(x+1)(x-2), \frac{1 \pm \sqrt{9}}{2}$

intercepts are (–1, 0) and (2, 0) (accept $x = -1$, $x = 2$) A1A1 N1N1

(b) evidence of appropriate method (M1)

eg $x_v = \frac{x_1 + x_2}{2}$, $x_v = -\frac{b}{2a}$, reference to symmetry

$x_v = 0.5$ A1 N2

[6]

3. (a) **METHOD 1**

$\ln(x+5) + \ln 2 = \ln(2(x+5))$ $(= \ln(2x+10))$ (A1)
interchanging x and y (seen anywhere) (M1)
eg $x = \ln(2y + 10)$
evidence of correct manipulation (A1)
eg $e^x = 2y + 10$

$f^{-1}(x) = \frac{e^x - 10}{2}$ A1 N2

METHOD 2

$y = \ln(x+5) + \ln 2$
$y - \ln 2 = \ln(x+5)$ (A1)
evidence of correct manipulation (A1)
eg $e^{y - \ln 2} = x + 5$
interchanging x and y (seen anywhere) (M1)
eg $e^{x - \ln 2} = y + 5$

$f^{-1}(x) = e^{x - \ln 2} - 5$ A1 N2

(b) **METHOD 1**

evidence of composition in correct order (M1)

eg $(g \circ f)(x) = g(\ln(x+5) + \ln 2)$

$= e^{\ln(2(x+5))} = 2(x+5)$

$(g \circ f)(x) = 2x + 10$ A1A1 N2

METHOD 2

evidence of composition in correct order (M1)
eg $(g \circ f)(x) = e^{\ln(x+5) + \ln 2}$
$= e^{\ln(x+5)} \times e^{\ln 2} = (x+5)\,2$
$(g \circ f)(x) = 2x + 10$ A1A1 N2

[7]

4. (a) **METHOD 1**

Using the discriminant $\Delta = 0$ (M1)

$k^2 = 4 \times 4 \times 1$

$k = 4$, $k = -4$ A1A1 N3

METHOD 2

Factorizing (M1)

$(2x \pm 1)^2$

$k = 4$, $k = -4$ A1A1 N3

(b) Evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ M1

eg $2(2\cos^2\theta - 1) + 4\cos\theta + 3$

$f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$ AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

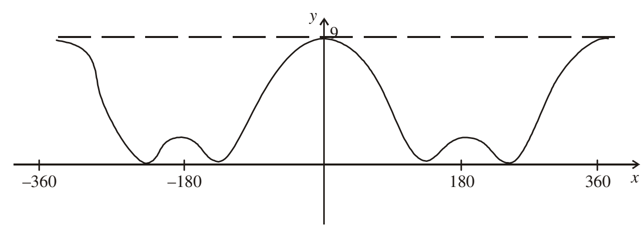
Attempting to solve for $\cos\theta$ M1

$\cos\theta = -\frac{1}{2}$ (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

METHOD 2

Sketch of $y = 4\cos^2\theta + 4\cos\theta + 1$ M1



Indicating 4 zeros (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

(d) Using sketch (M1)

$c = 9$ A1 N2

[11]

5. (a) $1 = A_0 e^{5k}$ A1

Attempt to find $\dfrac{dA}{dt}$ (M1)

eg $\dfrac{dA}{dt} = k A_0 e^{kt}$

Correct equation $0.2 = k A_0 e^{5k}$ A1

For any valid attempt to solve the system of equations M1

eg $\dfrac{0.2}{1} = \dfrac{k A_0 e^{5k}}{A_0 e^{5k}}$

$k = 0.2$ AG N0

(b) $100 = \dfrac{1}{e} e^{0.2t}$ A1

$t = \dfrac{\ln 100 + 1}{0.2}$ $(= 28.0)$ A1 N1

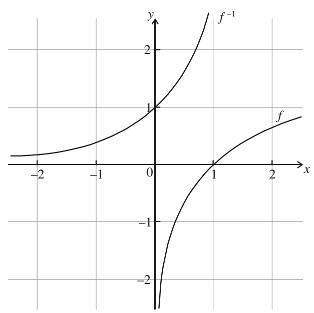
[6]

6. (a) (i) $f(a) = 1$ A1 N1

(ii) $f(1) = 0$ A1 N1

(iii) $f(a^4) = 4$ A1 N1

(b)



A1A1A1 N3

Note: *Award A1 for approximate reflection of f in y = x, A1 for y intercept at 1, and A1 for curve asymptotic to x axis.*

[6]

7. (a) Two correct factors A1A1

eg $y^2 + y - 12 = (y + 4)(y - 3)$, $(2^x)^2 + (2^x) - 12 = (2^x + 4)(2^x - 3)$

$a = 4$, $b = -3$ (or $a = -3$, $b = 4$) N2

(b) $2^x - 3 = 0$ (M1)

$2^x = 3$

$x = \dfrac{\ln 3}{\ln 2}$ $\left(\log_2 3, \dfrac{\log 3}{\log 2} \text{ etc.}\right)$ A1 N2

EITHER

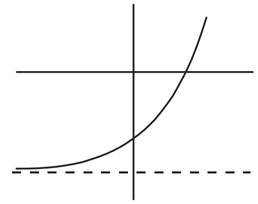
Considering $2^x + 4 = 0$ ($2^x = -4$) (may be seen earlier) A1

Valid reason R1 N1

eg this equation has no real solution, $2^x > 0$, graph does not cross the x-axis

OR

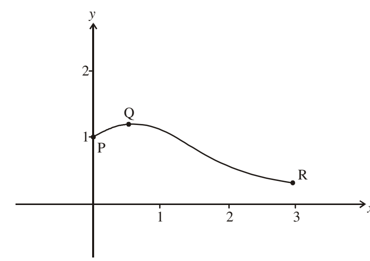
Considering graph of $y = 2^{2x} + 2^x - 12$ (asymptote does not need to be indicated) A1



There is only one point of intersection of the graph with x-axis. R1 N1

[6]

8. (a)



A1A1A1 N3

Note: *Award A1 for the shape of the curve, A1 for correct domain, A1 for labelling **both** points P and Q in approximately correct positions.*

(b) (i) Correctly finding derivative of $2x + 1$ *ie* 2 (A1)

Correctly finding derivative of e^{-x} *ie* $-e^{-x}$ (A1)

Evidence of using the product rule (M1)

$f'(x) = 2e^{-x} + (2x + 1)(-e^{-x})$ A1

$= (1 - 2x)e^{-x}$ AG N0

(ii) At **Q**, $f'(x) = 0$ (M1)

$x = 0.5$, $y = 2e^{-0.5}$ A1A1

Q is $(0.5, 2e^{-0.5})$ N3

(c) $1 \le k < 2e^{-0.5}$ A2 N2

(d) Using $f''(x) = 0$ at the point of inflexion M1

$e^{-x}(-3 + 2x) = 0$

This equation has only one root. R1

So f has only one point of inflexion. AG N0

(e) At R, $y = 7e^{-3}$ ($= 0.34850 \ldots$) (A1)

Gradient of (PR) is $\frac{7\mathrm{e}^{-3}-1}{3}\ (=-0.2172)$ (A1)

Equation of (PR) is $g(x) = \left(\frac{7\mathrm{e}^{-3}-1}{3}\right)x+1\ (=-0.2172x+1)$ A1

Evidence of appropriate method, involving subtraction of integrals or areas M2
Correct limits/endpoints A1

eg $\int_0^3 (f(x)-g(x))\,\mathrm{d}x$, area under curve − area under PR

Shaded area is $\int_0^3 \left((2x+1)\mathrm{e}^{-x} - \left(\frac{7\mathrm{e}^{-3}-1}{3}x+1\right)\right)\mathrm{d}x$

$= 0.529$ A1 N4

[21]

9. (a) For attempting to complete the square or expanding $y = 2(x-c)^2 + d$, or for showing the vertex is at (3, 5) M1

$y = 2(x-3)^2 + 5$ (accept $c = 3$, $d = 5$) A1A1 N2

(b) (i) $k = 2$ A1 N1

(ii) $p = 3$ A1 N1

(iii) $q = 5$ A1 N1

[6]

10. (a) $\mathrm{e}^{\ln(x+2)} = \mathrm{e}^3$ (M1)

$x + 2 = \mathrm{e}^3$ (A1)

$x = \mathrm{e}^3 - 2\ (= 18.1)$ A1 N3

(b) $\log_{10}(10^{2x}) = \log_{10} 500$ (accept lg and log for $\log_{10}$) (M1)

$2x = \log_{10} 500$ (A1)

$x = \frac{1}{2}\log_{10} 500 \quad \left(=\frac{\log 500}{\log 100} = 1.35\right)$ A1 N3

Note: *In both parts (a) and (b), if candidates use a graphical approach, award* ***M1*** *for a sketch, A1 for indicating appropriate points of intersection, and A1 for the answer.*

[6]

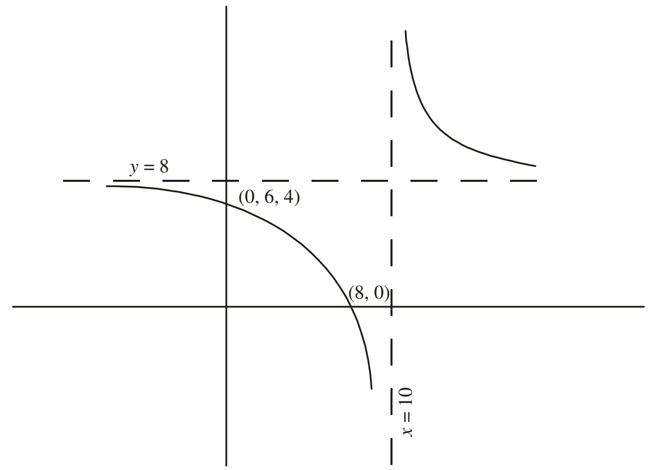
11. (a) (i) $x = 10$ (A1) (N1)

(ii) $y = 8$ (A1) (N1)

(b) (i) 6.4 (or (0, 6.4)) (A1) (N1)

(ii) 8 (or (8, 0)) (A1) (N1)

(c)



(A1)(A1)(A1)(A1) (N4)

Note: *Award (A1) for both asymptotes correctly drawn, (A1) for both intercepts correctly marked, (A1)(A1) for each branch drawn in approximately correct positions. Asymptotes and intercepts need not be labelled.*

(d) There is a vertical translation of 8 units.

(accept translation of $\begin{pmatrix}0\\8\end{pmatrix}$) (A2) (N2)

[10]