Exam Review **Algebra- Sequence, Series, Exponents, Logs, and Binomial (Markscheme)**

1. evidence of using binomial expansion (M1)

eg selecting correct term, $a^8b^0 + \binom{8}{1}a^7b + \binom{8}{2}a^6b^2 + \ldots$

evidence of calculating the factors, in any order A1A1A1

eg 56, $\frac{2^3}{3^3}, -3^5, \binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5$

$-4032x^3$ (accept $= -4030x^3$ to 3 sf) A1 N2

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2. (a) evidence of dividing two terms (M1)

eg $-\frac{1800}{3000}, -\frac{1800}{1080}$

$r = -0.6$ A1 N2

(b) evidence of substituting into the formula for the 10th term (M1)

eg $u_{10} = 3000(-0.6)^9$

$u_{10} = -30.2$ (accept the exact value -30.233088) A1 N2

(c) evidence of substituting into the formula for the infinite sum (M1)

$e.g.\ S = \frac{3000}{1.6}$

$S = 1875$ A1 N2

[6]

3. (a) $d = 3$ (A1)

evidence of substitution into $u_n = a + (n-1)d$ (M1)

eg $u_{101} = 2 + 100 \times 3$

$u_{101} = 302$ A1 N3

(b) correct approach (M1)

eg $152 = 2 + (n-1) \times 3$

correct simplification (A1)

eg $150 = (n-1) \times 3$, $50 = n - 1$, $152 = -1 + 3n$

$n = 51$ A1 N2

[6]

4. (a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$\left(e+\frac{1}{e}\right)^4 = \binom{4}{0}e^4 + \binom{4}{1}e^3\left(\frac{1}{e}\right) + \binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + \binom{4}{3}e\left(\frac{1}{e}\right)^3 + \binom{4}{4}\left(\frac{1}{e}\right)^4$

$\left(e+\frac{1}{e}\right)^4 = e^4 + 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 + 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4$

$\left(= e^4 + 4e^2 + 6 + \frac{4}{e^2} + \frac{1}{e^4}\right)$ N4

(b) $\left(e-\frac{1}{e}\right)^4 = e^4 - 4e^3\left(\frac{1}{e}\right) + 6e^2\left(\frac{1}{e}\right)^2 - 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4$

$\left(= e^4 - 4e^2 + 6 - \frac{4}{e^2} + \frac{1}{e^4}\right)$ (A1)

Adding gives $2e^4 + 12 + \frac{2}{e^4}$

$\left(\text{accept } 2\binom{4}{0}e^4 + 2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2 + 2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$ A1 N2

[6]

5. (a) 3, 6, 9 A1 N1

(b) (i) Evidence of using the sum of an AP M1

eg $\frac{20}{2}\ 2\times3+(20-1)\times3$

$\sum_{n=1}^{20} 3n = 630$ A1 N1

(ii) **METHOD 1**

Correct calculation for $\sum_{n=1}^{100} 3n$ (A1)

eg $\frac{100}{2}(2\times3+99\times3), 15150$

Evidence of subtraction (M1)

eg 15150 − 630

$\sum_{n=21}^{100} 3n = 14520$ A1 N2

METHOD 2

Recognising that first term is 63, the number of terms is 80 (A1)(A1)

eg $\frac{80}{2}(63+300),\ \frac{80}{2}(126+79\times3)$

$\sum_{n=21}^{100} 3n = 14520$ A1 N2[6]

6. Identifying the required term (seen anywhere) M1

eg $\binom{10}{8}\times2^2$

$\binom{10}{8} = 45$ (A1)

$4y^2$, 2×2, 4 (A2)

$a = 180$ A2 N4

[6]

7. (a) (i) $\log_c 15 = \log_c 3 + \log_c 5$ (A1)

$= p + q$ A1 N2

(ii) $\log_c 25 = 2\log_c 5$ (A1)

$= 2q$ A1 N2

(b) **METHOD 1**

$d^{\frac{1}{2}} = 6$ M1

$d = 36$ A1 N1

METHOD 2

For changing base M1

eg $\frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}$, $2\log_{10} 6 = \log_{10} d$

$d = 36$ A1 N1

[6]

8. (a) $\log_3 x - \log_3(x-5) = \log_3\left(\frac{x}{x-5}\right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

***Note:** If candidates have an incorrect or no answer to part (a) award (A1)(A0)*

if $\log\left(\frac{x}{x-5}\right)$ *seen in part (b).*

(b) **EITHER**

$\log_3\left(\frac{x}{x-5}\right) = 1$

$\frac{x}{x-5} = 3^1 (= 3)$ (M1)(A1)(A1)

$x = 3x - 15$

$-2x = -15$

$x = \frac{15}{2}$ (A1) (C4)

OR

$\frac{\log_{10}\left(\frac{x}{x-5}\right)}{\log_{10} 3} = 1$ (M1)(A1)

$\log_{10}\left(\frac{x}{x-5}\right) = \log_{10} 3$ (A1)

$x = 7.5$ (A1) (C4)

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