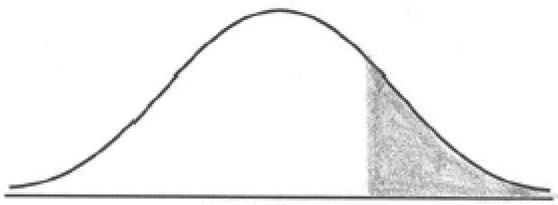
1. (a)



A1A1 N2

Note: *Award A1 for vertical line to right of mean, A1 for shading to right of* ***their*** *vertical line.*

(b) evidence of recognizing symmetry (M1)
e.g. 105 is one standard deviation above the mean so d is one standard deviation below the mean, shading the corresponding part,
$105 - 100 = 100 - d$

$d = 95$ A1 N2

(c) evidence of using complement (M1)
e.g. $1 - 0.32$, $1 - p$

$P(d < X < 105) = 0.68$ A1 N2

[6]

2. (a) (i) valid approach (M1)
e.g. np, $5 \times \frac{1}{5}$
$E(X) = 1$ A1 N2

(ii) evidence of appropriate approach involving binomial (M1)
e.g. $X \sim B\left(5, \frac{1}{5}\right)$

recognizing that Mark needs to answer 3 **or more** questions correctly (A1)
e.g. $P(X \geq 3)$

valid approach M1
e.g. $1 - P(X \leq 2)$, $P(X = 3) + P(X = 4) + P(X = 5)$

P(pass) = 0.0579 A1 N3

(b) (i) evidence of summing probabilities to 1 (M1)
e.g. $0.67 + 0.05 + (a + 2b) + ... + 0.04 = 1$

some simplification that clearly leads to required answer
e.g. $0.76 + 4a + 2b = 1$ A1

$4a + 2b = 0.24$ AG N0

(ii) correct substitution into the formula for expected value (A1)
e.g. 0(0.67) + 1(0.05) + ... + 5(0.04)

some simplification (A1)
e.g. $0.05 + 2a + 4b + ... + 5(0.04) = 1$

correct equation A1
e.g. $13a + 5b = 0.75$

evidence of solving (M1)

$a = 0.05$, $b = 0.02$ A1A1 N4

(c) attempt to find probability Bill passes (M1)
e.g. $P(Y \geq 3)$

correct value 0.19 A1
Bill (is more likely to pass) A1 N0

[17]

3. (a) $E(X) = 2$ A1 N1

(b) evidence of appropriate approach involving binomial (M1)
e.g. $\binom{10}{3}(0.2)^3$, $(0.2)^3(0.8)^7$, $X \sim B(10, 0.2)$

$P(X = 3) = 0.201$ A1 N2

(c) **METHOD 1**

$P(X \leq 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133$ (= 0.87912...) (A1)
evidence of using the complement (seen anywhere) (M1)
e.g. 1 – any probability, $P(X > 3) = 1 - P(X \leq 3)$

$P(X > 3) = 0.121$ A1 N2

METHOD 2

recognizing that $P(X > 3) = P(X \geq 4)$ (M1)
e.g. summing probabilities from $X = 4$ to $X = 10$

correct expression or values (A1)
e.g. $\sum_{r=4}^{10}\binom{10}{r}(0.2)^{10-r}(0.8)^r$

0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001

$P(X > 3) = 0.121$ A1 N2

[6]

4. (a) evidence of approach (M1)

e.g. finding 0.84…, using $\frac{23.7-21}{\sigma}$

correct working (A1)

e.g. $0.84... = \frac{23.7-21}{\sigma}$, graph

$\sigma = 3.21$ A1 N2

(b) (i) evidence of attempting to find $P(X < 25.4)$ (M1)

e.g. using $z = 1.37$

$P(X < 25.4) = 0.915$ A1 N2

(ii) evidence of recognizing symmetry (M1)

e.g. $b = 21 - 4.4$, using $z = -1.37$

$b = 16.6$ A1 N2

[7]

5. (a) evidence of using binomial probability (M1)

e.g. $P(X = 2) = \binom{7}{2}(0.18)^2(0.82)^5$

$P(X = 2) = 0.252$ A1 N2

(b) **METHOD 1**

evidence of using the complement M1

e.g. $1 - (P(X \le 1))$

$P(X \le 1) = 0.632$ (A1)

$P(X \ge 2) = 0.368$ A1 N2

METHOD 2

evidence of attempting to sum probabilities M1

e.g. P(2 heads) + P(3 heads) + ... + P(7 heads), 0.252 + 0.0923 + …

correct values for each probability (A1)

e.g. 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061

$P(X \ge 2) = 0.368$ A1 N2

[5]

6. ***Note:*** *Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.*

$W \sim N(2.5, 0.3^2)$

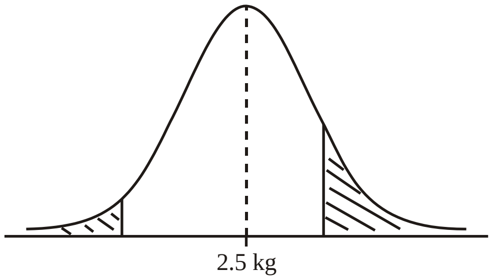
(a) (i) $z = -1.67$ (accept 1.67) (A1)

$P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) A1 N2

(ii) $z = 1$ (A1)

$P(W > 2.8) = 0.159$ A1 N2

(iii)



A1A1 N2

Note: *Award A1 for a vertical line to left of mean andshading to left, A1 for vertical line to right of mean and shading to right.*

(iv) Evidence of appropriate calculation M1

eg $1 - (0.047790 + 0.15866)$, $0.8413 - 0.0478$

$P = 0.7936$ AG N0

Note: *The final value may vary depending on what level of accuracy is used.*

Accept their value in subsequent parts.

(b) (i) $X \sim B(10, 0.7935...)$

Evidence of calculation M1

eg $P(X = 10) = (0.7935...)^{10}$

$P(X = 10) = 0.0990$ (3 sf) A1 N1

(ii) **METHOD 1**

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

$P(X \le 6) = 0.1325...$ (or $P(X = 1) + ... + P(X = 6)$) (A1)

evidence of using the complement (M1)

eg $P(X \ge 7) = 1 - P(X \le 6)$, $P(X \ge 7) = 1 - P(X < 7)$

$P(X \ge 7) = 0.867$ A1 N3

METHOD 2

Recognizing $X \sim B(10, 0.7935...)$ (may be seen in (i)) (M1)

For adding terms from $P(X = 7)$ to $P(X = 10)$ (M1)

$P(X \ge 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$ (A1)

$= 0.867$ A1 N3

[13]

7. $X \sim N(\mu, \sigma^2)$, $P(X < 3) = 0.2$, $P(X > 8) = 0.1$

$P(X < 8) = 0.9$ (M1)

Attempt to set up equations (M1)

$\frac{3-\mu}{\sigma} = -0.8416$, $\frac{8-\mu}{\sigma} = 1.282$ A1A1

$3 - \mu = -0.8416\sigma$

$8 - \mu = 1.282\sigma$

$5 = 2.1236\sigma$

$\sigma = 2.35, \quad \mu = 4.99$ A1A1 N4

[6]

8. (a) (i) $a = -1$ (A1)

$b = 0.5$ (A1)

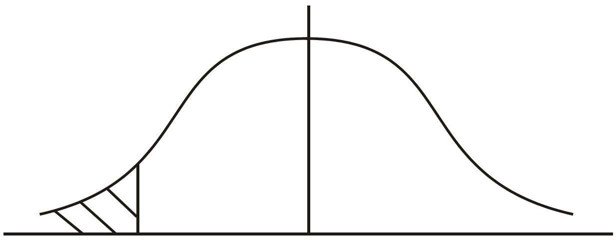
(ii) (a) 0.841 (A2)

(b) $0.6915 - 0.1587$ (or $0.8413 - 0.3085$) (M1)

$= 0.533$ (3 sf)

(A1) (N2) 6

(b) (i) Sketch of normal curve (A1)(A1)



(ii) $c = 0.647$ (A2) 4

[10]

9. (a) $P(M \geq 350) = 1 - P(M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)$ (M1)

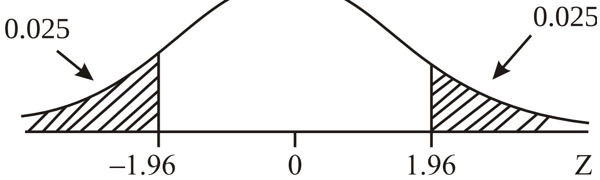
$= 1 - P(Z < 1.333) = 1 - 0.9088$

$= 0.0912$ (accept 0.0910 to 0.0920) (A1)

OR

$P(M \geq 350) = 0.0912$ (G2)

(b)



$P(Z < 1.96) = 1 - 0.025 = 0.975$ (A1)

$1.96\ (30) = 58.8$ (M1)

$310 - 58.8 < M < 310 + 58.8 \Rightarrow a = 251, b = 369$ (A1)

OR

$251 < M < 369$ (G3)

Note: *Award (G1) if only one of the end points is correct.*

[5]

10. ***Note:*** *Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf*

(a) $z = \frac{197 - 187.5}{9.5} = 1.00$ (M1)

P (Z > 1) = 1 – Φ(1) = 1 – 0.8413 = 0.1587

= 0.159 (3 sf) (A1)

= 15.9% (A1)

OR

P (H > 197) = 0.159 (G2)

= 15.9% (A1) 3

(b) Finding the 99th percentile

Φ(a) = 0.99 => a = 2.327 (accept 2.33) (A1)

=> 99% of heights under 187.5 + 2.327(9.5) = 209.6065 (M1)

= 210 (3 sf) (A1)

OR

99% of heights under 209.6 = 210 cm (3 sf) (G3)

Height of standard doorway = 210 + 17 = 227 cm (A1) 4

[7]

11. (a) $Z = \frac{25 - 25.7}{0.50} = -1.4$ (M1)

P(Z < –1.4) = 1 – P(Z < 1.4)

= 1 – 0.9192

= 0.0808 (A1)

OR

P(W < 25) = 0.0808 (G2) 2

(b) P(Z < –a) = 0.025 ⇒ P(Z < a) = 0.975

⇒ a = 1.960 (A1)

$\frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96\,(0.50)$ (M1)

= 25 + 0.98 = 25.98 (A1)

= 26.0 (3 sf) (AG)

OR

$\frac{25.0 - 26.0}{0.50} = -2.00$ (M1)

P(Z < –2.00) = 1 – P(Z < 2.00)

= 1 – 0.9772 = 0.0228 (A1)

≈ 0.025 (A1)

OR

$\mu = 25.98$ (G2)

$\Rightarrow$ mean = 26.0 (3 sf) (A1)(AG) 3

(c) Clearly, by symmetry $\mu = 25.5$ (A1)

$Z = \dfrac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$ (M1)

$\Rightarrow \sigma = 0.255$ kg (A1) 3

(d) On average, $\dfrac{\text{cement saving}}{\text{bag}} = 0.5$ kg (A1)

$\dfrac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = \$0.40$ (M1)

To save \$5000 takes $\dfrac{5000}{0.40} = 12500$ bags (A1) 3

[11]

12. (a) Area $A = 0.1$ (A1) 1

(b) **EITHER** Since $p\ (X \geq 12) = p\ (X \leq 8)$, (M1)

then 8 and 12 are symmetrically disposed around the mean. (M1)(R1)

Thus mean $= \dfrac{8+12}{2}$ (M1)

$= 10$ (A1)

Notes: *If a candidate says simply "by symmetry $\mu = 10$" with no further explanation award [3 marks] (M1, A1, R1). As a full explanation is requested award an additional (A1) for saying since p(X < 8) = p(X > 12) and another (A1) for saying that the normal curve is symmetric.*

OR $p\ (X \geq 12) = 0.1 \Rightarrow p\left(Z \geq \dfrac{12-\mu}{\sigma}\right) = 0.1$ (M1)

$\Rightarrow p\left(Z \leq \dfrac{12-\mu}{\sigma}\right) = 0.9$

$p\ (X \leq 8) = 0.1 \Rightarrow p\left(Z \leq \dfrac{8-\mu}{\sigma}\right) = 0.1$

$\Rightarrow p\left(Z \leq \dfrac{\mu-8}{\sigma}\right) = 0.9$ (A1)

So $\dfrac{12-\mu}{\sigma} = \dfrac{\mu-8}{\sigma}$ (M1)

$\Rightarrow 12 - \mu = \mu - 8$ (M1)

$\Rightarrow \mu = 10$ (A1) 5

(c) $\Phi\left(\frac{12-10}{\sigma}\right) = 0.9$ (A1)(M1)(A1)

Note: *Award (A1) for* $\left(\frac{12-10}{\sigma}\right)$*, (M1) for standardizing, and (A1) for 0.9.*

$\Rightarrow \frac{2}{\sigma} = 1.282$ (or 1.28) (A1)

$\sigma = \frac{2}{1.282}\left(\text{or } \frac{2}{1.28}\right)$ (A1)

= 1.56 (3 sf) (AG) 5

Note: *Working backwards from $\sigma = 1.56$ to show it leads the given data should receive a maximum of [3 marks] if done correctly.*

(d) $p\ (X \le 11) = p\left(Z \le \frac{11-10}{1.561}\right)$ (or 1.56) (M1)(A1)

Note: *Award (M1) for standardizing and (A1) for* $\left(\frac{11-10}{1.561}\right)$.

= $p\ (Z \le 0.6407)$ (or 0.641 or 0.64) (A1)
= $\Phi(0.6407)$ (M1)
= 0.739 (3 sf) (A1) 5

[16]

13. (a) p (4 heads) = $\binom{8}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{8-4}$ (M1)

$= \frac{8\times7\times6\times5}{1\times2\times3\times4}\times\left(\frac{1}{2}\right)^8$

$= \frac{70}{256} \cong 0.273$ (3 sf) (A1) 2

(b) p (3 heads) = $\binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3} = \frac{8\times7\times6}{1\times2\times3}\times\left(\frac{1}{2}\right)^8$

$= \frac{56}{256} \cong 0.219$ (3 sf) (A1) 1

(c) p (5 heads) = p (3 heads) (by symmetry) (M1)
p (3 or 4 or 5 heads) = p (4) + 2p (3) (M1)

$= \frac{70+2\times56}{256} = \frac{182}{256}$

≈ 0.711 (3 sf) (A1) 3

[6]