1. Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing P(X > 105).

(2)

(b) Given that P($X < d$) = P(X > 105), find the value of d.

(2)

(c) Given that P(X > 105) = 0.16 (correct to two significant figures), find P($d < X$ < 105).

(2)

(Total 6 marks)

2. A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find E(X).

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

y	0	1	2	3	4	5
P($Y = y$)	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

(ii) Given that E(Y) = 1, find a and b.

(8)

(c) Find which student is more likely to pass the test.

(3)

(Total 17 marks)

3. A multiple choice test consists of ten questions. Each question has five answers. Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly.

(1)

(b) Find the probability that Jose answers exactly three questions correctly.

(2)

(c) Find the probability that Jose answers more than three questions correctly.

(3)

(Total 6 marks)

4. The scores of a test given to students are normally distributed with a mean of 21. 80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores.

(3)

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of b.

(4)

(Total 7 marks)

5. The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(2)

(b) Find the probability of obtaining **at least** two heads.

(3)

(Total 5 marks)

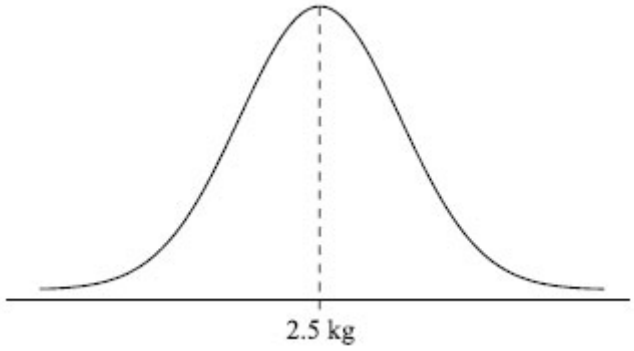
6. The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) Copy the diagram below. Shade the areas that represent the probabilities from parts (i) and (ii).



(iv) **Hence** show that the probability that it weighs between 2 kg and 2.8 kg is 0.7936 (to four significant figures).

(7)

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg.

(6)

(Total 13 marks)

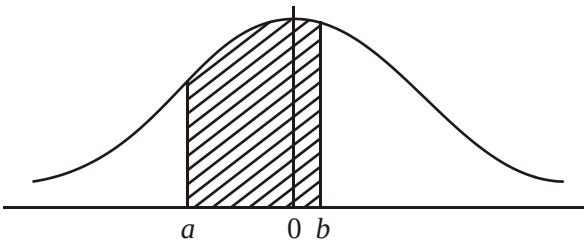
7. The heights of certain flowers follow a normal distribution. It is known that 20% of these flowers have a height less than 3 cm and 10% have a height greater than 8 cm.

Find the value of the mean μ and the standard deviation σ.

(Total 6 marks)

8. Reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds.

(a) The graph below is that of the **standard** normal curve. The shaded area represents the probability that the reaction time of a person chosen at random is between 0.70 and 0.79 seconds.



(i) Write down the value of a and of b.

(ii) Calculate the probability that the reaction time of a person chosen at random is

(a) greater than 0.70 seconds;

(b) between 0.70 and 0.79 seconds.

(6)

Three percent (3%) of the population have a reaction time less than c seconds.

(b) (i) Represent this information on a diagram similar to the one above. Indicate clearly the area representing 3%.

(ii) Find c.

(4)

(Total 10 marks)

9. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

(2)

(b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b.

(3)

(Total 5 marks)

10. In a country called *Tallopia,* the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm?

(3)

(b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm.

(4)

(Total 7 marks)

11. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg?

(2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg.

(3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements.

(3)

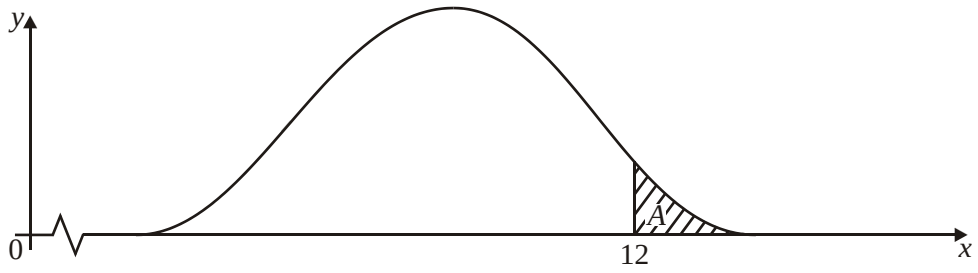
The cost of the new machine is \$5000. Cement sells for \$0.80 per kg.

(d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment?

(3)

(Total 11 marks)

12. The graph shows a normal curve for the random variable X, with mean μ and standard deviation σ.



It is known that $p(X \geq 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A.

(1)

It is also known that $p(X \leq 8) = 0.1$.

(b) Find the value of μ, explaining your method in full.

(5)

(c) Show that $\sigma = 1.56$ to an accuracy of three significant figures.

(5)

(d) Find $p(X \leq 11)$.

(5)

(Total 16 marks)

13. A fair coin is tossed eight times. Calculate

(a) the probability of obtaining exactly 4 heads;

(2)

(b) the probability of obtaining exactly 3 heads;

(1)

(c) the probability of obtaining 3, 4 or 5 heads.

(3)

(Total 6 marks)