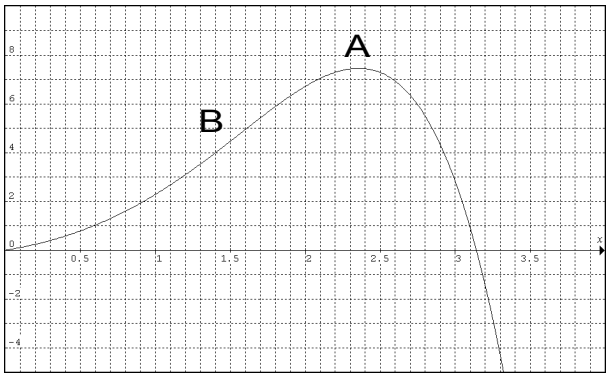
Calculus Review - Part 2

Curve Properties

The graph of $f(x) = e^x \sin x$ is shown for $0 \leq x \leq 4$ with a maximum at the point A and an inflection point at B.



a) Find *f'(x)*.

b) Hence, find the exact coordinates of the maximum, *A*.

c) Find *f''(x)* in simplified form.

d) Hence, show that A is a maximum.

e) Find the x-coordinate of the point B.

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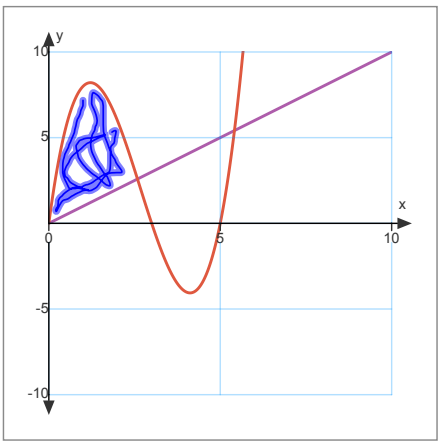
Ex.) Write the equation of the asymptotes of $y = 2 + \frac{3}{x-1}$.

Ex.) Describe the behaviour of $f(x) = \frac{3x^2 - x}{2x^2 + 1}$ for large values of *x*.

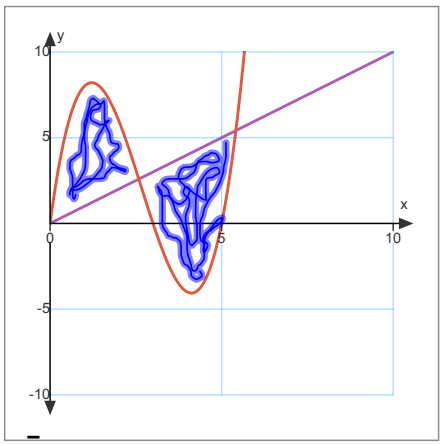
Ex.) Consider the functions $f(x) = x$ and $g(x) = x(x-3)(x-5)$.

a) Write an expression for the shaded area.

b) Find the shaded area. (assume GDC can be used)



Ex.) Repeat the exercise above for the shaded area below.



Ex.) Given that $f''(x) = 2x - 2$ and *f(x)* has a minimum at (3, -7), find the equation of *f(x)*.

Ex.) a) Write the equation of the normal to $y = (x-3)e^x$ at x = 1

b) Find the coordinates where the normal cuts *f(x)* again. (assume GDC may be used)

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Applications to Kinematics

Displacement - s(t)

Differentiate ↓ ↑ Integrate

Velocity - v(t)

Differentiate ↓ ↑ Integrate

Acceleration - a(t)

Don't forget your constant of integration ("+ c") when integrating.

Ex.) The velocity function of a particle is given by $v(t) = 3t^2 - e^{2t} + \frac{1}{t}$ ms^{-1} for $t \geq 1$ sec. When t=3 sec., the particle is 5 m right of the origin.

a) Find the displacement when t=4 sec.

b) Find the acceleration when t=4 sec.

Optimization

Steps:

1) Draw a clear diagram

2) Construct an equation with the variable to be optimized (cost, volume, area, etc.). Make sure that all quantities are expressed in terms of this variable.

3) Find the min/max using either:
   i) GDC
   ii) first derivative and (ideally) a sign table
   iii) second derivative test

Ex.) Anji has decided to create a rectangular pen. She wants a total area of 30 m^2. The material required for 3 of the sides cost $4 per metre and the material required for the front costs $9 per metre.

Write an equation for the cost of the materials and find the dimensions that minimize cost.

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