

Calculus

The derivative function is given by $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

The derivative of some function $f(x)$ is written $f'(x)$ and gives the slope of the tangent at any point. This slope of the tangent corresponds to the "instantaneous rate of change" of the curve. Wherever $f'(x)=0$, we may have a maximum, a minimum, or a point of inflection. Sign tables or the second derivative test help us determine which is occurring.

The second derivative of some function $f(x)$ is written $f''(x)$ and tells us about the concavity of the curve at any point. If f'' is negative at some point, the original curve is concave down (_____). If f'' is positive at some point, the original curve is concave up (_____). An inflection point indicates a change in concavity. At an inflection point, $f''(a)=0$ **and** $f''(x)$ changes signs around this point.

Second Derivative Test

If, at some x-value, a , on a curve:

- $f'(a)=0$ and $f''(a)<0$ (ie. the curve is concave down), we have a max

- $f'(a)=0$ and $f''(a)>0$ (ie. the curve is concave up), we have a min

Ex.) Given $f(x)$, find $f'(x)$.

a) $f(x) = 4x^{\frac{3}{2}} - 3\ln x - e^{4x+1} + \cos x + 2$ b) $f(x) = x \sin x$

c) $f(x) = 3x\sqrt{x}$

d) $f(x) = \sin^2(3x)$

e) $f(x) = \frac{5x^2 - 3x + 2}{x}$

f) $f(x) = \frac{4x^2 - 1}{x + 3}$

g) $f(x) = \ln\left(\frac{2x - 1}{3x^2 + 1}\right)$

h) $f(x) = (2x^3 - 3x)^5$

—

Ex.) Find the following indefinite integrals.

a) $\int \left(3x^5 - \cos x + \frac{3}{x} \right) dx$

b) $\int e^{3x-1} dx$

c) $\int 5 dx$

d) $\int (x \cos x^2) dx$

Ex.) Find the following definite integrals.

a) $\int_{-2}^3 (3x^2 - 1) dx$

b) $\int_0^{\pi} (e^x + \sin 2x) dx$

Volumes of Revolution

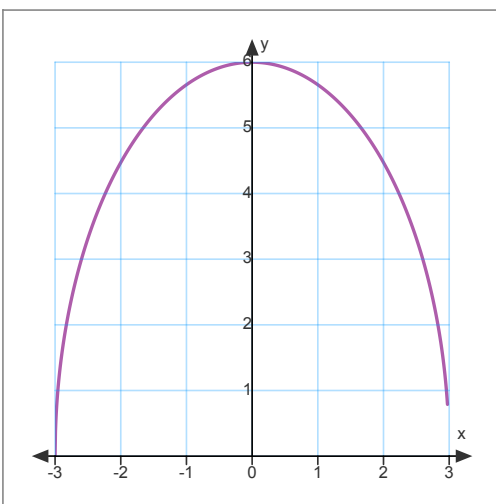
For some function $y = f(x)$ the volume created by rotating the function about the x-axis over some interval $[a, b]$ is given by:

$$V = \pi \int_a^b (f(x))^2 dx$$

Given in your formula booklet as:

$$V = \int_a^b \pi y^2 dx$$

Ex.) Find the volume of the solid formed by rotating the curve $y = 2\sqrt{9 - x^2}$ about the x-axis.



—