

Exam Review

Calculus

1. Attempting to integrate. (M1)
 $y = x^3 - 5x + c$ (A1)(A1)(A1)
 substitute (2, 6) to find c ($6 = 2^3 - 5(2) + c$) (M1)
 $c = 8$ (A1)
 $y = x^3 - 5x + 8$ (Accept $x^3 - 5x + 8$) (C6)

[6]

2. (a) $x = \frac{1}{5}$ or $5x - 1 = 0$ (A1) (N1) 1

- (b) $f'(x) = \frac{(5x-1)(6x)-(3x^2)(5)}{(5x-1)^2}$ (M1)(A1)
 $= \frac{30x^2 - 6x - 15x^2}{(5x-1)^2}$ (may be implied) (A1)
 $= \frac{15x^2 - 6x}{(5x-1)^2}$ (accept $a = 15, b = -6$) (A1) (N2) 4

[5]

3. (a) (i) intersection points $x = 3.77, x = 8.30$ (may be seen as the limits) (A1)(A1)
 approach involving subtraction and integrals (M1)
 fully correct expression A2

eg $\int_{3.77}^{8.30} ((-4 \cos(0.5x) + 2) - (\ln(3x-2) + 1)) dx,$
 $\int_{3.77}^{8.30} g(x) dx - \int_{3.77}^{8.30} f(x) dx$ N5

- (ii) $A = 6.46$ (A1) (N1)

- (b) (i) $f'(x) = \frac{3}{3x-2}$ A1A1 (N2)

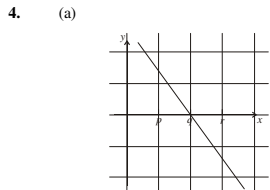
Note: Award A1 for numerator (3), A1 for denominator (3x-2), but penalize 1 mark for additional terms.

- (ii) $g'(x) = 2 \sin(0.5x)$ A1A1 (N2)

Note: Award A1 for 2, A1 for sin (0.5x), but penalize 1 mark for additional terms.

- (c) evidence of using derivatives for gradients (M1)
 correct approach (A1)
 eg $f'(x) = g'(x)$, points of intersection
 $x = 1.43, x = 6.10$ A1A1 (N2)N2

[14]



- A1A1 (N2)

1

Note: Award A1 for negative gradient throughout, A1 for x- intercept of q. It need not be linear.

(b)

	x - coordinate
(i) Maximum point on f	r
(i) Inflection point on f	q

A1 (N1)

A1 (N1)

(c) **METHOD 1**

Second derivative is zero, second derivative changes sign.

R1R1 (N2)

METHOD 2

There is a maximum on the graph of the first derivative.

R2 (N2)

[6]

5. (a) evidence of factorizing 3/division by 3 (A1)

eg $\int_1^5 3f(x) dx = 3 \int_1^5 f(x) dx, \frac{12}{3}, \int_1^5 \frac{3f(x) dx}{3}$

(do not accept 4 as this is show that)

evidence of stating that reversing the limits changes the sign

A1

eg $\int_5^1 f(x) dx = - \int_1^5 f(x) dx$

$\int_5^1 f(x) dx = -4$

AG (N0)

- (b) evidence of correctly combining the integrals (seen anywhere) (A1)

eg $I = \int_1^2 (x + f(x)) dx + \int_2^5 (x + f(x)) dx = \int_1^5 (x + f(x)) dx$

evidence of correctly splitting the integrals (seen anywhere)

(A1)

eg $I = \int_1^5 x dx + \int_1^5 f(x) dx$

$\int x dx = \frac{x^2}{2}$ (seen anywhere) (A1)

$\int_1^5 x dx = \left[\frac{x^2}{2} \right]_1^5 = \frac{25}{2} - \frac{1}{2} \left(= \frac{24}{2}, 12 \right)$ (A1)

$I = 16$

A1 (N3)

[7]

6. (a) $f'(x) = x^2 + 4x - 5$ A1A1A1 (N3)

- (b) evidence of attempting to solve $f'(x) = 0$ (M1)
 evidence of correct working (A1)

eg $(x+5)(x-1), \frac{-4 \pm \sqrt{16+20}}{2}$, sketch

$x = -5, x = 1$

(A1)

so $x = -5$

A1 (N2)

(c) **METHOD 1**

$f''(x) = 2x + 4$ (may be seen later)

evidence of setting second derivative = 0 (M1)

eg $2x + 4 = 0$

$x = -2$ A1

N2

METHOD 2

2

evidence of use of symmetry
eg midpoint of max/min, reference to shape of cubic
correct calculation

eg $\frac{-5+1}{2}$,

$x = -2$

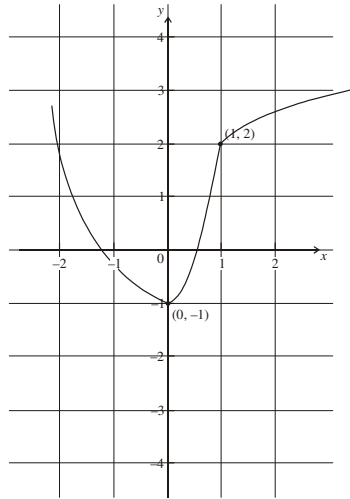
(d) attempting to find the value of the derivative when $x = 3$
 $f'(3) = 16$

valid approach to finding the equation of a line
eg $y - 12 = 16(x - 3)$, $12 = 16 \times 3 + b$
 $y = 16x - 36$

(M1)
A1
A1 N2
(M1)
A1
M1
A1 N2

[14]

7.



Notes: On interval $[-2, 0]$, award A1 for decreasing, A1 for concave up.
On interval $[0, 1]$, award A1 for increasing, A1 for concave up.
On interval $[1, 2]$, award A1 for change of concavity, A1 for concave down.

A1A1A1A1A1A1 N6

8. (a) $\frac{dy}{dx} = 3 \cos 3x$

A1 N1

(b) $\frac{dy}{dx} = \frac{x}{\cos^2 x} + \tan x$ accept $x \sec^2 x + \tan x$

A1A1 N2

(c) **METHOD 1**

Evidence of using the quotient rule

(M1)

$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - \ln x}{x^2}$

A1A1

[6]

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

N3

METHOD 2

$y = x^{-1} \ln x$

Evidence of using the product rule

(M1)

$\frac{dy}{dx} = x^{-1} \times \frac{1}{x} + \ln x(-1)(x^{-2})$

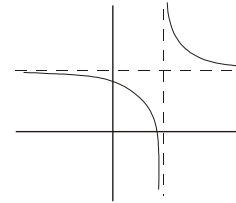
A1A1

$\frac{dy}{dx} = \frac{1}{x^2} - \frac{\ln x}{x^2}$

N3

[6]

9. (a)



Notes: Award A1 for both asymptotes shown. The asymptotes need not be labelled. Award A1 for the left branch in approximately correct position, A1 for the right branch in approximately correct position.

A1A1A1 N3

(b) (i) $y = 3, x = \frac{5}{2}$ (must be equations)

A1A1 N2

(ii) $x = \frac{14}{6} \left(\frac{7}{3} \text{ or } 2.33, \text{ also accept } \left(\frac{14}{6}, 0 \right) \right)$

A1 N1

(iii) $y = \frac{14}{6} \left(y = 2.8 \right) \left(\text{accept } \left(0, \frac{14}{5} \right) \text{ or } (0, 2.8) \right)$

A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2} \right) dx = 9x + 3 \ln(2x-5) - \frac{1}{2(2x-5)} + C$

A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$

(M1)

Correct expression

A1

$eg \int_3^a \pi \left(3 + \frac{1}{2x-5} \right)^2 dx, \pi \int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2} \right) dx,$
 $\left[9x + 3 \ln(2x-5) - \frac{1}{2(2x-5)} \right]_3^a$

$$\text{Substituting } \left(9a + 3\ln(2a-5) - \frac{1}{2(2a-5)}\right) - \left(27 + 3\ln 1 - \frac{1}{2}\right)$$

Setting up an equation

$$9a - \frac{1}{2(2a-5)} - 27 + \frac{1}{2} + 3\ln(2a-5) - 3\ln 1 = \left(\frac{28}{3} + 3\ln 3\right)$$

Solving gives $a = 4$

A1

(M1)

A1 N2

[17]

10. $f'(x) = 12x^2 + 2$

When $x = 1, f(1) = 6$ (seen anywhere)

When $x = 1, f'(1) = 14$

Evidence of taking the negative reciprocal

eg $\frac{-1}{14}x, \frac{1}{-14}, -0.0714$

Equation is $y - 6 = -\frac{1}{14}(x-1) \left(y = -\frac{1}{14}x + \frac{85}{14}, y = -0.0714x + 6.07\right)$

A1A1

(A1)

(A1)

(M1)

A1 N4

[6]

11. (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$)

A1A1 N2

(ii) $x = 3$ (must be an equation)

A1 N1

(b) $y = (x-1)(x-5)$

$= x^2 - 6x + 5$

(A1)

$= (x-3)^2 - 4$ (accept $h = 3, k = -4$)

A1A1 N3

(c) $\frac{dy}{dx} = 2(x-3) (= 2x-6)$

A1A1 N2

(d) When $x = 0, \frac{dy}{dx} = -6$

(A1)

$y - 5 = -6(x-0)$ ($y = -6x + 5$ or equivalent)

A1 N2

[10]

12. (a) Evidence of using $a = \frac{dy}{dt}$

(M1)

eg $3e^{3t-2}$

$a(1) = 3e$ ($= 8.15$)

A1 N2

(b) Attempt to solve $e^{3t-2} = 22.3$

(M1)

eg $(3t-2)(\ln e) = \ln 22.3$, sketch

$t = 1.70A1$

N2

(c) Evidence of using $s = \int v dt$ (limits not required)

M1

e.g. $\int e^{3t-2} dt, \frac{1}{3} [e^{3t-2}]_0^1$

$\frac{1}{3}(e^1 - e^{-2}) \left[= \frac{1}{3}(e - e^{-2}) = 0.861 \right]$

A1 N1

[6]

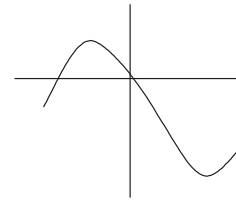
13. (a) EITHER

Recognizing that tangents parallel to the x -axis mean maximum and minimum (may be seen on sketch)
Sketch of graph of f

R1

M1

5



OR

Evidence of using $f'(x) = 0$

M1

Finding $f'(x) = 3x^2 - 6x - 24$

A1

$3x^2 - 6x - 24 = 0$

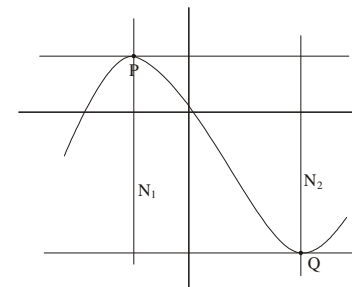
Solutions $x = -2$ or $x = 4$

THEN

Coordinates are $P(-2, 29)$ and $Q(4, -79)$

A1A1 N1N1

(b)



(i) $(4, 29)$

A1 N1

(ii) $(-2, -79)$

A1 N1

[6]

14. Using $V = \int \pi y^2 dx$

(M1)

Correctly integrating $\int \left(\frac{1}{x^2}\right)^2 dx = \frac{x^2}{2}$

A1

$V = \pi \left[\frac{x^2}{2} \right]_0^a$

A1

$= \frac{\pi a^2}{2}$

(A1)

Setting up their equation $\left(\frac{1}{2}\pi a^2 = 0.845\pi\right)$

M1

$a^2 = 1.69$

$a = 1.3$ A1

N2

[6]

6