1. The curve $y = f(x)$ passes through the point (2, 6).

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 5$, find y in terms of x.

(Total 6 marks)

2. Let $f(x) = \frac{3x^2}{5x-1}$.

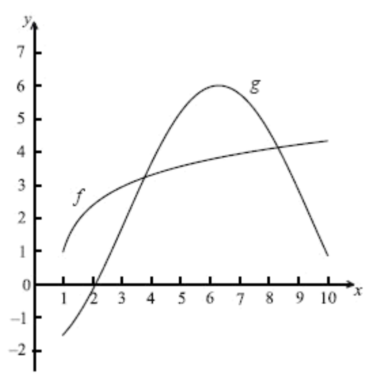
(a) Write down the **equation** of the vertical asymptote of $y = f(x)$.

(1)

(b) Find $f'(x)$. Give your answer in the form $\frac{ax^2 + bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$.

(4)
(Total 5 marks)

3. The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



(a) Let A be the area of the region **enclosed** by the curves of f and g.

(i) Find an expression for A.

(ii) Calculate the value of A.

(6)

(b) (i) Find $f'(x)$.

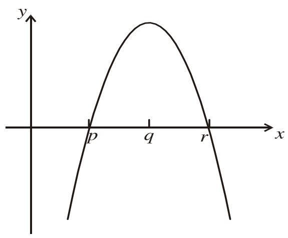
(ii) Find $g'(x)$.

(4)

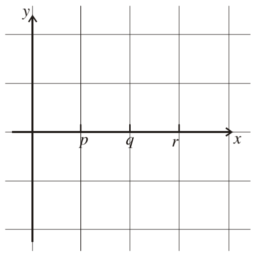
(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x.

(4)
(Total 14 marks)

4. The diagram below shows part of the graph of the **gradient** function, $y = f'(x)$.



(a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x-intercept.



(2)

(b) Complete the table, for the graph of $y = f(x)$.

	x-coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

(2)

(c) Justify your answer to part (b) (ii).

(2)
(Total 6 marks)

5. Let $\int_1^5 3f(x)\,\mathrm{d}x = 12$.

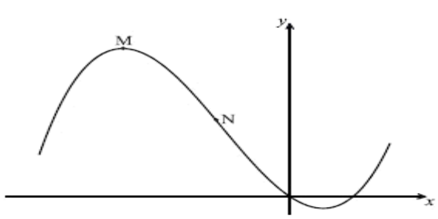
(a) Show that $\int_1^5 f(x)\,\mathrm{d}x = -4$.

(2)

(b) Find the value of $\int_1^5 (x + f(x))\,\mathrm{d}x + \int_2^5 (x + f(x))\,\mathrm{d}x$.

(5)
(Total 7 marks)

6. Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$.

(3)

(b) Find the x-coordinate of M.

(4)

(c) Find the x-coordinate of N.

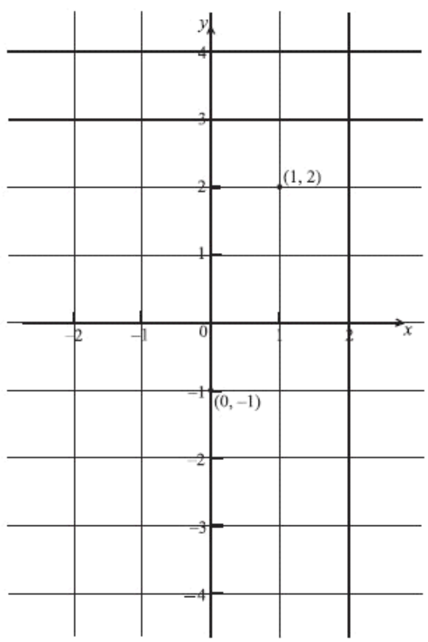
(3)

(d) The line L is the tangent to the curve of f at (3, 12). Find the equation of L in the form $y = ax + b$.

(4)
(Total 14 marks)

7. On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	-1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



(Total 6 marks)

8. Differentiate each of the following with respect to x.

(a) $y = \sin 3x$

(1)

(b) $y = x \tan x$

(2)

(c) $y = \dfrac{\ln x}{x}$

(3)
(Total 6 marks)

9. The function $f(x)$ is defined as $f(x) = 3 + \dfrac{1}{2x-5}$, $x \neq \dfrac{5}{2}$.

(a) Sketch the curve of f for $-5 \leq x \leq 5$, showing the asymptotes.

(3)

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept.

(4)

(c) The region enclosed by the curve of f, the x-axis, and the lines $x = 3$ and $x = a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

(i) Find $\displaystyle\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) \mathrm{d}x$.

(ii) Hence, given that $V = \pi\left(\dfrac{28}{3} + 3\ln 3\right)$, find the value of a.

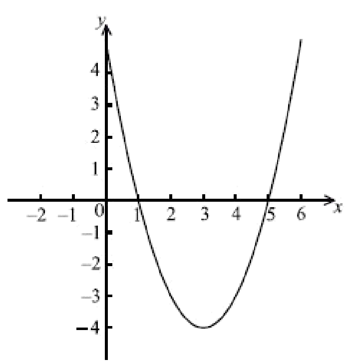
(10)

(Total 17 marks)

10. Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

(Total 6 marks)

11. The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q;

(ii) the equation of the axis of symmetry of the curve.

(3)

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(3)

(c) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(d) Let T be the tangent to the curve at the point (0, 5). Find the equation of T.

(2)
(Total 10 marks)

12. The velocity, $\boldsymbol{v}$, in m s^{-1} of a particle moving in a straight line is given by $v = e^{3t-2}$, where t is the time in seconds.

(a) Find the acceleration of the particle at $t = 1$.

(b) At what value of t does the particle have a velocity of 22.3 m s^{-1}?

(c) Find the distance travelled in the first second.

(Total 6 marks)

13. Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

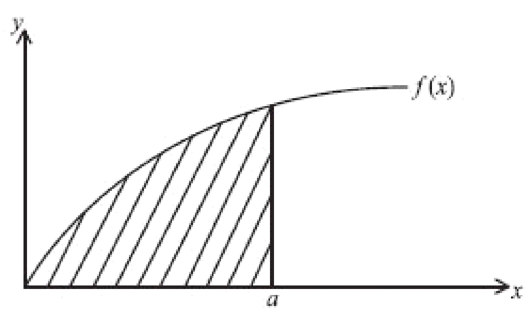
(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1.

(Total 6 marks)

14. The shaded region in the diagram below is bounded by $f(x) = \sqrt{x}$, $x = a$, and the x-axis. The shaded region is revolved around the x-axis through 360°. The volume of the solid formed is 0.845π.



Find the value of a.

(Total 6 marks)