

Exam Review

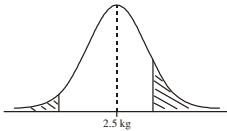
Probability (Markscheme)

1. (a) (i) $P(B) = \frac{3}{4}$ A1 N1
- (ii) $P(R) = \frac{1}{4}$ A1 N1
- (b) $p = \frac{3}{4}$ A1 N1
- $s = \frac{1}{4}, t = \frac{3}{4}$ A1 N1
- (c) (i) $P(X = 3)$
- $= P(\text{getting 1 and 2}) = \frac{1}{4} \times \frac{3}{4}$ A1
- $= \frac{3}{16}$ AG N0
- (ii) $P(X = 2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4} \left(\text{or } 1 - \frac{3}{16} \right)$ (A1)
- $= \frac{13}{16}$ A1 N2
- (d) (i)
- | | | |
|------------|-----------------|----------------|
| X | 2 | 3 |
| $P(X = x)$ | $\frac{13}{16}$ | $\frac{3}{16}$ |
- (ii) evidence of using $E(X) = \sum xP(X = x)$ (M1)
- $E(X) = 2 \left(\frac{13}{16} \right) + 3 \left(\frac{3}{16} \right)$ (A1)
- $= \frac{35}{16} \left(= 2 \frac{3}{16} \right)$ A1 N2
- (e) win \$10 \Rightarrow scores 3 one time, 2 other time (M1)
- $P(3) \times P(2) = \frac{13}{16} \times \frac{3}{16}$ (seen anywhere) A1
- evidence of recognizing there are different ways of winning \$10 (M1)
- eg $P(3) \times P(2) + P(2) \times P(3), 2 \left(\frac{13}{16} \times \frac{3}{16} \right)$
- $\frac{36}{256} + \frac{3}{256} + \frac{36}{256} + \frac{3}{256}$
- $P(\text{win } \$10) = \frac{78}{256} \left(= \frac{39}{128} \right)$ A1 N3

2. **Note:** Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.

$W \sim N(2.5, 0.3^2)$

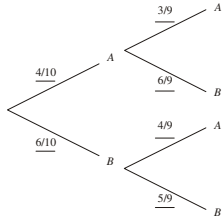
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- (a) (i) $z = -1.67$ (accept 1.67) (A1)
- $P(W < 2) = 0.0478$ (accept answers between 0.0475 and 0.0485) A1 N2
- (ii) $z = 1$ (A1)
- $P(W > 2.8) = 0.159$ A1 N2
- (iii)
- 
- Note:** Award A1 for a vertical line to left of mean and shading to left, A1 for vertical line to right of mean and shading to right. A1A1 N2
- (iv) Evidence of appropriate calculation (M1)
- eg $1 - (0.047790 + 0.15866), 0.8413 - 0.0478$
- $P = 0.7936$ AG N0
- Note:** The final value may vary depending on what level of accuracy is used. Accept their value in subsequent parts.
- (b) (i) $X \sim B(10, 0.7935\dots)$ (M1)
- Evidence of calculation (M1)
- eg $P(X = 10) = (0.7935\dots)^{10}$
- $P(X = 10) = 0.0990$ (3 sf) A1 N1
- (ii) **METHOD 1**
- Recognizing $X \sim B(10, 0.7935\dots)$ (may be seen in (i)) (M1)
- $P(X \leq 6) = 0.1325\dots$ (or $P(X = 1) + \dots + P(X = 6)$) (A1)
- evidence of using the complement (M1)
- eg $P(X \geq 7) = 1 - P(X \leq 6), P(X \geq 7) = 1 - P(X < 7)$
- $P(X \geq 7) = 0.867$ A1 N3
- METHOD 2**
- Recognizing $X \sim B(10, 0.7935\dots)$ (may be seen in (i)) (M1)
- For adding terms from $P(X = 7)$ to $P(X = 10)$ (M1)
- $P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030$ (A1)
- $= 0.867$ A1 N3
3. (a) For summing to 1 (M1)
- eg $0.1 + a + 0.3 + b = 1$
- $a + b = 0.6$ A1 N2
- (b) evidence of correctly using $E(X) = \sum x f(x)$ (M1)
- eg $0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b, 0.1 + a + 0.6 + 3b = 1.5$
- Correct equation $0 + a + 0.6 + 3b = 1.5$ ($a + 3b = 0.9$) (A1)
- Solving simultaneously gives
- $a = 0.45 \quad b = 0.15$ A1A1 N3

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4. (a)



$$(b) \left(\frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{6}{10} \times \frac{4}{9}\right)$$

$$= \frac{48}{90} \left(\frac{8}{15}, 0.533\right)$$

5. (a) $\frac{19}{120} (=0.158)$

(b) $35 - (8 + 5 + 7) (= 15)$

$$\text{Probability} = \frac{15}{120} \left(= \frac{3}{24} = \frac{1}{8} = 0.125 \right)$$

(c) Number studying = 76

Number not studying = 120 - number studying = 44

$$\text{Probability} = \frac{44}{120} \left(= \frac{11}{30} = 0.367 \right)$$

6. (a) $P(F \cup S) = 1 - 0.14 (= 0.86)$

Choosing an appropriate formula

eg $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Correct substitution

eg $P(F \cap S) = 0.93 - 0.86$

$P(F \cap S) = 0.07$

Notes: There are several valid approaches. Award (A1)(M1)A1 for relevant working using any appropriate strategy eg formula, Venn Diagram, or table.

Award no marks for the incorrect solution

$P(F \cap S) = 1 - P(F) + P(S) = 1 - 0.93 = 0.07$

(b) Using conditional probability

eg $P(F|S) \left(= \frac{P(F \cap S)}{P(S)} \right)$

$$P(F|S) = \frac{0.07}{0.62}$$

$= 0.113$

(c) F and S are **not** independent

EITHER

If independent $P(F|S) = P(F)$, $0.113 \neq 0.31$

OR

If independent $P(F \cap S) = P(F)P(S)$, $0.07 \neq 0.31 \times 0.62 (= 0.1922)$

(d) Let $P(F) = x$

$P(S) = 2P(F) (= 2x)$

For independence $P(F \cap S) = P(F)P(S) (= 2x^2)$

Attempt to set up a quadratic equation

eg $P(F \cup S) = P(F)P(S) - P(F)P(S)$, $0.86 = x + 2x - 2x^2$

$2x^2 - 3x + 0.86 = 0$

$x = 0.386$, $x = 1.11$

$P(F) = 0.386$

7. $X \sim N(\mu, \sigma^2)$, $P(X < 3) = 0.2$, $P(X > 8) = 0.1$

$P(X < 8) = 0.9$

Attempt to set up equations

$$\frac{3 - \mu}{\sigma} = -0.8416, \quad \frac{8 - \mu}{\sigma} = 1.282$$

$3 - \mu = -0.8416\sigma$

$8 - \mu = 1.282\sigma$

$5 = 2.1236\sigma$

$\sigma = 2.35$, $\mu = 4.99$

8. (a) For attempting to use the formula $P(E \cap F) = P(E)P(F)$

Correct substitution or rearranging the formula

eg $\frac{1}{3} = \frac{2}{3} P(F)$, $P(F) = \frac{P(E \cap F)}{P(E)}$, $P(F) = \frac{1}{3}$

$P(F) = \frac{1}{2}$

(b) For attempting to use the formula $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$P(E \cup F) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3}$$

$= \frac{5}{6} (= 0.833)$

9. (a) $X \sim B(100, 0.02)$
 $E(X) = 100 \times 0.02 = 2$

(b) $P(X = 3) = \binom{100}{3} (0.02)^3 (0.98)^{97}$

$= 0.182$

A1A1A1 N3

M1M1

A1 N1

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A1 N1

(M1)

A1 N2

(A1)

(M1)

A1 N3

[6]

(A1)

(M1)

A1

AG N0

(M1)

(A1)

A1 N3

A1 N1

R1R1 N2

R1R1 N2

(A1)

(R1)

(M1)

A2

(A1)

(A1) N5

[16]

(M1)

(M1)

A1A1

A1A1 N4

[6]

(M1)

A1

A1 N2

(M1)

A1

A1 N2

[6]

A1 1

(M1)

A1 2

(c) **METHOD 1**

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) && \text{M1} \\ &= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99}) && \text{(M1)} \\ &= 0.597 && \text{A1} \quad 2 \end{aligned}$$

METHOD 2

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) && \text{(M1)} \\ &= 1 - 0.40327 && \text{(A1)} \\ &= 0.597 && \text{A1} \quad 2 \end{aligned}$$

Note: Award marks as follows for finding $P(X > 1)$, if working shown.

$$\begin{aligned} P(X \geq 1) &&& \text{A0} \\ &= 1 - P(X < 2) = 1 - 0.67668 && \text{M1(ft)} \\ &= 0.323 && \text{A1(ft)} \quad 2 \end{aligned}$$

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10. (a) Independent (I) (C2)
(b) Mutually exclusive (M) (C2)
(c) Neither (N) (C2)

Note: Award part marks if the candidate shows understanding of I and/or

M

eg I $P(A \cap B) = P(A)P(B)$ (M1)

M $P(A \cup B) = P(A) + P(B)$ (M1)

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