1 **a** $\tan x = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

b $\cos(x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$= \frac{5\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{2}, \frac{5\pi}{6}$

2 **a** $\cos^2 A = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$

$\sin^2 A = 1 - \cos^2 A = 2\sqrt{3} - 3$

b $\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$

$= \frac{2\sqrt{3}-3}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}} = \frac{(2\sqrt{3}-3)(4+2\sqrt{3})}{16-12}$

$= \frac{8\sqrt{3}+12-12-6\sqrt{3}}{4} = \frac{2\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{2}$

3 **a** $45° = \frac{\pi}{4}$

$P = (2 \times 8) + (8 \times \frac{\pi}{4}) = 22.3$ cm

b area of sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi$

area of triangle $= \frac{1}{2} \times 8^2 \times \sin\frac{\pi}{4}$

$= 32 \times \frac{1}{\sqrt{2}} = 16\sqrt{2}$

area of segment $= 8\pi - 16\sqrt{2}$

$= 8(\pi - 2\sqrt{2})$ cm^2

4 $2\sin^2\theta + \sin\theta - (1 - \sin^2\theta) = 2$

$3\sin^2\theta + \sin\theta - 3 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1+36}}{6}$

$\sin\theta = \frac{1}{6}(-1+\sqrt{37})$

or $\frac{1}{6}(-1-\sqrt{37})$ [no solutions]

$\theta = 57.9, 180 - 57.9$

$\theta = 57.9°, 122.1°$

5 $3\sin^2 x + 4\sin x - 4 = 0$

$(3\sin x - 2)(\sin x + 2) = 0$

$\sin x = \frac{2}{3}$ or -2 [no solutions]

$x = 0.73, \pi - 0.7297$

$x = 0.73^c, 2.41^c$

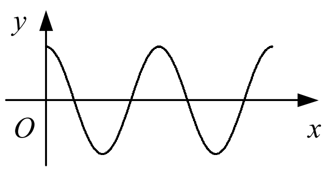
6 $2\sin x = 3\cos x$

$\tan x = 1.5$

$x = 0.98, \pi + 0.9828 = 0.98, 4.12$

$\therefore (0.98, 1.66), (4.12, -1.66)$

7 **a**



b $2x = 180 - 60, 180 + 60,$

$540 - 60, 540 + 60$

$= 120, 240, 480, 600$

$x = 60, 120, 240, 300$

8 $12\cos^2\theta = 7\sin\theta$

$12(1 - \sin^2\theta) = 7\sin\theta$

$12\sin^2\theta + 7\sin\theta - 12 = 0$

$(4\sin\theta - 3)(3\sin\theta + 4) = 0$

$\sin\theta = 0.75$ or $-\frac{4}{3}$ [no solutions]

$\theta = 48.6, 180 - 48.6$

$\theta = 48.6, 131.4$

9 **a** $\tan 15° = \frac{\sqrt{3}-1}{1+(\sqrt{3}\times 1)} = \frac{\sqrt{3}-1}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3}$

$= -\tfrac{1}{2}(\sqrt{3}-3-1+\sqrt{3})$

$= 2 - \sqrt{3}$

b $\tan 345° = -\tan 15° = \sqrt{3} - 2$

10 $(1 - \cos^2 x) + 5\cos x - 3\cos^2 x = 2$

$4\cos^2 x - 5\cos x + 1 = 0$

$(4\cos x - 1)(\cos x - 1) = 0$

$\cos x = 0.25$ or 1

$x = 75.5,\ 360 - 75.5$ or $0,\ 360$

$x = 0,\ 75.5°$ (1dp), $284.5°$ (1dp), $360°$

11 **a** $\angle ABC = 180 - (41 + 26) = 113$

$\frac{BC}{\sin 41} = \frac{18}{\sin 113}$

$BC = \frac{18\times\sin 41}{\sin 113} = 12.8$ cm

b $= \tfrac{1}{2}\times 18\times 12.829\times\sin 26$

$= 50.6\text{ cm}^2$

12 $6\cos^2\theta + 5\cos\theta - 4 = 0$

$(3\cos\theta + 4)(2\cos\theta - 1) = 0$

$\cos\theta = 0.5$ or $-\tfrac{4}{3}$ [no solutions]

$\theta = 60,\ 360 - 60$

$\theta = 60°,\ 300°$

13 $\sin^2 x + 5\sin x = 2(1 - \sin^2 x)$

$3\sin^2 x + 5\sin x - 2 = 0$

$(3\sin x - 1)(\sin x + 2) = 0$

$\sin x = \tfrac{1}{3}$ or -2 [no solutions]

$x = 19.5,\ 180 - 19.5$

$x = 19.5°,\ 160.5°$

14 **a** LHS $= (1 - \cos^2\theta)^2 - 2(1 - \cos^2\theta)$

$= 1 - 2\cos^2\theta + \cos^4\theta - 2 + 2\cos^2\theta$

$= \cos^4\theta - 1$

= RHS

b LHS $= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

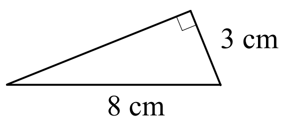
$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

= RHS

15 **a**



$\cos(\angle PQR) = \tfrac{3}{8}$

$\therefore\ \angle PQR = 1.186^c$

b $RS^2 = 8^2 - 3^2 = 55$

$RS = \sqrt{55} = 7.42$ cm (3sf)

c obtuse $\angle SPU = 2\times\angle PQR = 2.3728$

reflex $\angle RQT = 2\pi - \angle SPU = 3.9104$

length of rubber band

$= (2\times 7.4162) + (2\times 2.3728) + (5\times 3.9104)$

$= 39.1$ cm (3sf)