C2 SEQUENCES AND SERIES — Worksheet F

1 The first and fourth terms of a geometric series are 108 and 32 respectively.

a Find the third term of the series. **(5)**

b Find the sum to infinity of the series. **(2)**

2 Expand $(1 - 2x)^5$ in ascending powers of x, simplifying each coefficient. **(4)**

3 An internet site has 3600 subscribers at the start of a promotional campaign.

In a model of the results of the campaign, it is assumed that the site will gain 200 new subscribers in the first week and that in subsequent weeks the number of new subscribers will be 15% greater each week.

a Show that, according to this model, the site will gain 304 new subscribers in the fourth week of the campaign. **(2)**

b Find the total number of subcribers to the site predicted by the model after ten weeks of the campaign, assuming that no subscriptions are cancelled in this period. **(5)**

4 **a** Find the first three terms in the expansion of $(1 + 4x)^7$ in ascending powers of x. **(3)**

b Hence, find the coefficient of x^2 in the expansion of

$$(1 + 2x)^2(1 + 4x)^7.$$ **(3)**

5 **a** Write down the first four terms in the expansion in ascending powers of x of $(1 + \frac{x}{k})^{2n}$, where k is a non-zero constant, n is an integer and $n > 1$. **(4)**

Given that the coefficient of x^3 is half the coefficient of x^2,

b show that $3k = 4(n - 1)$. **(3)**

Given also that the coefficient of x is 2,

c find the values of n and k. **(3)**

6 The second and third terms of a geometric series are $\sqrt{6}$ and $3\sqrt{2}$ respectively.

a Find, in surd form, the first term and the common ratio of the series. **(4)**

b Show that the sum of the first eight terms of the series is $40\sqrt{2}(\sqrt{3} + 1)$. **(4)**

7 Evaluate $$\sum_{r=1}^{9} (3^r - 1).$$ **(5)**

8 **a** Find the first four terms in the expansion of $(1 + 2x)^9$ in ascending powers of x. **(4)**

b Show that, if terms involving x^4 and higher powers of x may be ignored,

$$(1 + 2x)^9 + (1 - 2x)^9 = 2 + 288x^2.$$ **(3)**

c Hence find the value of

$$1.002^9 + 0.998^9,$$

giving your answer to 7 significant figures. **(2)**

9 Given that

$$(k - x)^9 \equiv a - bx + bx^2 + \ldots,$$

find the values of the positive integers a, b and k. **(7)**

10 Expand $(3 + 2x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

11 The first term of a geometric series is t and the sum to infinity of the series is $3t$.

a Find the common ratio of the series. **(3)**

Given also that the sum of the first four terms of the series is 130,

b find the value of t. **(4)**

12 **a** Expand $(1 - 2x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Hence, or otherwise, find the coefficient of y^2 in the expansion of

$$(1 + 4y - 2y^2)^4.$$ **(4)**

13 A company buys a new car for £12 000 at the start of one year. In a model, it is assumed that each year the value of a car decreases by 25% of its value at the start of that year.

a Show that the value of the car after four years is £3800 to 3 significant figures. **(2)**

The company plans to buy one new car for £12 000 at the start of each subsequent year.

b Using the same model, find the total value of all the cars the company will have bought under this plan immediately after the purchase of the eighth car. **(5)**

14 The polynomial p(x) is defined by

$$\text{p}(x) = (x + 3)^4 - (x + 1)^4.$$

a Show that $(x + 2)$ is a factor of p(x). **(2)**

b Fully factorise p(x). **(7)**

c Hence show that there is only one real solution to the equation p(x) = 0. **(3)**

15 $$\text{f}(x) \equiv (1 - x)(1 + 2x)^n, \quad n \in \mathbb{N}.$$

Given that the coefficient of x^2 in the binomial expansion of f(x) is 198, find

a the value of n, **(6)**

b the coefficient of x^3 in the expansion. **(3)**

16 Expand $(\frac{3}{x} - x)^4$ in descending powers of x, simplifying the coefficient in each term. **(4)**

17 The sum, S_n, of the first n terms of a series is given by

$$S_n = 3^n - 1.$$

a Show that the fourth term of the series is 54. **(3)**

b Show that the nth term of the series can be expressed in the form $k(3^n)$ where k is an exact fraction to be found. **(4)**

c Prove that the series is geometric. **(3)**

18 An arithmetic series has first term 3, second term x and fourth term y.

a Find an expression for y in terms of x. **(3)**

Given also that 3, x and y are the first, second and fourth terms respectively of a geometric series,

b show that $x^3 - 27x + 54 = 0$, **(4)**

c by first finding a linear factor of $x^3 - 27x + 54$, find the two possible values of x. **(6)**